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Regular (Landline) Phone Company versus Cellular Phone Company: The Non-Cooperative Case

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ABSTRACT

In this paper we developed a model that has two important perspectives, theoretical and practical. From the theoretical point of view we develop a model of two different rival companies supplying different communication services whose objectives are profit maximization. Although the services somewhat overlap, they are certainly not complete substitutes. In some sense the services are substitutes but in another sense they can also be complements.
These factors may lead to several optimal pricing policies of coordination and cooperation between the two companies. These results differ from the classical pricing policy of the well known regular duopoly case. The use of comparative static analysis have led to some straightforward results, however some other results are surprising, counter intuitive and in some sense even provocative.

Keywords: *Network industry, Cellular and landline pricing, Duopoly, Substitute, Comparative static analysis*

1. INTRODUCTION

The rapid worldwide spread of cellular phones over the last two decades has generated serious research amongst economists as to the degree of market penetration, demand growth, and the saturation level of this new product in terms of the number of instruments per capita (see for example Tishler et al [1]). Some of those recent studies deal with estimating and forecasting the demand for cellular services as a function of its own price and various socio-economic variables while ignoring the fact that cell phones are penetrating a market where an old communication device already exists—namely the landline phone. There is no doubt that there is a large area of overlap between the cell and the landline phone and for certain uses they can be looked at as substitute goods. If so, the cross-price elasticity should be taken into account.

In a very recent paper Reece [2] discussed the demand for cellular phone services as a function of the degree of mobility of the society, such as the use of the mobile service in cars, the number of drivers per household, etc. Moreover, Reece presents evidence that cellular and landline services are substitutes to a certain extent. This interesting observation can be restated by saying that although there is a measure of substitutability between these instruments they certainly cannot be considered full or complete substitutes. By this we mean that the degree of substitution between landline and cellular services is clearly not a one to one conversion, since landline is more suitable for home and office while unsuitable for use in cars, flights, and general travel where only cellular services are appropriate.

If by law and upon regulatory policy two different and separate companies supply each type of service, we have a duopoly environment where there is some degree of substitutability between the two goods, but the goods are not complete substitutes. A simple duopoly pricing model of the Nash type would not be sufficient in this case. Furthermore, in the simple model of Nash no cooperation exists while in our model we do consider the possibility of some degree of cooperation. We investigate the effects on the pricing policies of the two rival companies of factors such as the degree of substitution between the two services, the cross elasticity of demand between them, the degree of cooperation between the two rival companies, etc.

In recent years several studies of this kind were undertaken (see for example Sung and Lee [3]) where the degree of substitution and competition between these two products were examined empirically. They show the inverse effects between quantities demand of cellular and landline phones. These results are very similar to American evidence published very recently by Blumberg and Luke [4] where it appears from preliminary results of the NHIS (National Health Interview Survey) for January-June 2007 that more than one out of every eight American homes (13.6%) had cellular phones during the first half of 2007. The evidence appears to support the predictions of Hausman [5], [6], who claims that over time more and more families are switching over to cell phones alone, and are using their cell phones as a full substitute for the landline. Still and all, the vast majority of the public are not yet ready to give up their landline.

In another paper, Hausman [7] talks about the increase of consumer welfare resulting from the entry and penetration of this new product, which is based on new communication technology. He wonders how society can establish the value added of these new products and services as a result of the entry into the market.

Our target in the current paper is different: We concentrate more on the profit effects and pricing policy of both competitors as a function of the entry of the cell phone into the communication market.

In Miravete and Roller [8] the non-linear pricing determined by the duopoly is introduced for the specific case where both firms are offering two kinds of services. The two services are offered by the two firms are either complements or substitutes. The willingness of consumers to purchase from each firm is affected by the pricing decision of the competing firm.

In our model we deal with a simple (single) pricing of a two firm industry. Each firm offers one service (phone calls) while the two kinds of services are substitutes to a certain extent. In our model we deal with imperfect substitutability but not with the case of perfect substitutability (see Vives [9]).

We do however argue that due to a degree of substitutability, price determination by each firm is inter-connected. However, in our case we also assumed that some degree of cooperation between firms exists due to their sharing revenues from cross calls agreements. This system of sharing revenues (partial or sometimes full sharing) has recently become popular in the communication markets in several countries, since it increases the number of successful contacts, thus increases revenues to all "rival suppliers".

The purpose of our paper is to explore the possibility of generating a successful connection between the sender and the receiver that depends on at least two of the following factors:

(i) The readiness of the receiver to accept the communication signal and to respond to the call of the sender, i.e., the sender is willing to carry out the call with the sender, since he/she initiated it. Otherwise he/she would not connect, but the receiver who identifies the caller by simple modern devices who makes the connection may not respond (or answer) to the connection/call.

(ii) The availability of the receiver, i.e., he/she may not be available to answer a phone call because, for example, the call is to his/her home while he/she is at work. This may specially occur when the receiver uses a regular phone with simple wire/line system. However, the fast development of technology since the last decades and the sophisticated cellular mobile phones generate a much easier and efficient connectivity system use of voicemail or answering voice machine etc between sender and receiver, including SMS messages,. The degree of availability nowadays is much greater and more efficient where the receiver can be approached if he/she desires almost anywhere in the world at any time, and at any circumstances that one can possibly imagine. This improves welfare implications and Pareto improvement process for consumers and producers, as well as senders and/or simultaneously receivers in the communication market. These improvements may reflect different pricing policy applied either by the monopoly profit-maximizing supplier of the communication market, or by a social planner or competitive supplier, working in oligopoly environments.

The entry of the cellular industry into the communication network industry is very unique in the sense that it may be considered as a technological improvement of the regular landline phones. The latter are phones that in order to send and/or receive a message the customer should be located at a specific time in a specific locations at the end of a "connectivity line". Otherwise the calls cannot be accomplished. The cellular generates innovation in that the phone call can be accomplished *more often* (although not necessarily always) in much broader environments. Still the new technology of cellular communication is not a complete substitute for landline phones. In some cases the new technology might indeed be a full substitute for the old technology which may be almost totally eliminated or excluded from the market. Many items that were used in the past are vanished, where new innovations lead to the exclusion of that items, e.g., ice refrigerator disappeared when electric refrigeration was introduced a few generations ago, or handy manual typewriters were eliminated by the introduction of electronic typewriters that in the last decade almost disappeared as a result of computers. Motor cars substituted last century's use of horses and donkeys as means of transportation in the Western world.

In the case of the cellular phone vs. landline phone, however, this kind of phenomenon does not occur, since the two items are not fully substituted and many qualifications exist in both or only in one of the two instruments. Thus, the question we should raise is what are the welfare and profit implications on the society and the original phone company (like AT&T, etc) upon the entry of the cellular device.

The social and profit effect in the case of full substitution of items (e.g., donkey vs. motor car, etc) are different from the scenario where a newly introduced item is attached to an old item, i.e., when the services are not fully substituted. This concept is what motives us to work on this issue.

The structure of this paper is as follows: In the next section we develop the general model, followed by comparative static analysis, then numerical example and lastly implications and conclusions section.

2. THE MODEL

Assume two companies supplying communication services of calls between senders and receivers, regular phone calls (landline) and cellular phone calls that are substitutes in calls supplying services, however, the two instruments are not full substitutes, since they are used in different timing and location, and the qualifications of services are not identical. Thus, we assume four simple linear demand functions q_{ij} as follows:

We denote q_{LL} to illustrate the demand for phone calls from sender to receiver both using landline phones.

(1) $$q_{LL} = A_1 - P_{LL}$$

P_{LL} represents the call price via landline.

We denote q_{LC} to show the demand for phone calls from landline to cellular phone.

(2) $$q_{LC} = A_2 - P_{LC} + fP_{LL}$$

P_{LC} represents the call price from a landline phone to a cellular phone.

q_{CC} shows the demand for calls from cellular to cellular phone

(3) $$q_{CC} = B_1 - P_{CC}$$

where P_{CC} is the price of a call from cellular to cellular phone.

q_{CL} represents the demand for calls from cellular to landline phone.

(4) $$q_{CL} = B_2 - P_{CL} + gP_{CC}$$

P_{CL} represents the call price from cellular to landline phone

The above coefficients hold true for the following conditions:

$0 < f,\ g < 1$ indicating cross-price sensitivity are smaller than the direct price sensitivity on quantities of call demands from landline to cellular phone and vice versa.

We assume further that the cross-call prices, i.e., the price of a call from landline to cellular phone and vice versa, P_{LC} and P_{CL} hold the following:

(5) $$P_{LC} = P_{LL} + \alpha P_{CC} \qquad 0 < \alpha < 1$$

(6) $$P_{CL} = P_{CC} + \beta P_{LL} \qquad 0 < \beta < 1$$

The above equations indicate the idea that the prices charged for cross call cover compensation to both suppliers of the connection services.

Let us verify our point now denoting equations (5) and (6). P_{CC} represents the full price for each call from cellular to cellular phone and P_{LL} represents the full price for each call using landline to landline phone service which the companies charge their customers. However, the cellular phone company partially charges its competitor (the landline phone company) for using their phone service, and the same applies vice versa for the landline phone company who also partially charges its rival, i.e., the cellular phone company, since α and β are positive but less than 1.

Furthermore, the two companies supply the cross calls to their customers and charge them an extra fee only according to what they are charged by their competitor. This means that both companies get a "flat-rate" price on all calls made by their customers, regardless of whether the calls are from landline to landline or from cellular to cellular, or cross-calls between cellular to landline and vice versa.

Another assumption of the model is that the regular (landline) calls can be accomplished only at some fraction of the day, t where $0 < t < 1$, since the customers (either the receiver or the sender) are not always available to receive the calls. However, in the case of the cellular phone we assume for simplicity that it can be accomplished 24-hour a day, since customers can often be reached in different locations, timings and circumstances, and are not bounded to attach to specific timings and locations to receive or send calls successfully a less restrictive assumption could be that cellular phone is completed T hours a day where $t < T < 24$. Therefore, the caller sender from a landline phone may try to reach the receiver via his/her landline phone. If the receiver is not reachable by the regular phone at a portion of (1-t) the landline sender can call his mobile phone. In the opposite case, a sender from a cellular phone can always reach the receiver on his cellular phone, but again he can reach the receiver on his landline phone only at t proportion of the day.[1]

Another assumption indicates that revenues can be gained only if the calls are carried out. However, costs are incurred when the receiver tries to call or send a message, even if the calls cannot be accomplished. Furthermore, we assume constant-marginal cost of C_{LL} that is the cost per call from line to line and C_{CC} that is the cost per call from cellular to cellular. However, the costs of cross-calls are higher by some proportions that are defined as follows:

(7) $$C_{LC} = (1+\gamma)\, C_{LL}$$

(8) $$C_{CL} = (1+\delta)\, C_{CC}$$

where $\gamma, \delta > 0$

The last assumption we include in our model concerns revenues shared by two companies from cross-calls, from landline to cellular phones and vice versa. We

[1] There are still enough reasons to call from a cellular to a landline phone, since both are not full substitutes and several services are only available on landline.

assume that the sender's phone company from where the call is originated, receives *h* of the revenues from cross-calls and the receiver's phone company gets the remainder (1-*h*), where 0<*h*<1.

This can be justified based on both practical and theoretical grounds. We do know that the bills of customers show charges that the sending company charges for itself as well as charges based on payments to other servers for cross calls.

Based on the above assumptions, we can define the profit functions of a regular landline phone company, Π_L, and a cellular phone company, Π_C, as follows:

(9)

$$\Pi_L = [t P_{LL}(A_1 - P_{LL})] + [(1-t)(P_{LL} + h\alpha P_{CC})\cdot(A_2 + f\cdot P_{LL} - P_{LL-} - \alpha P_{CC})] + [t(1-h)\beta P_{LL}(B_2 + gP_{CC} - P_{CC} - \beta P_{LL})] - [C_{LL}\cdot(A_1 - P_{LL}) + (1+\gamma)C_{LL}\cdot(A_2 + f P_{LL} - P_{LL} - \alpha P_{CC})]$$

(10)

$$\Pi_C = [P_{CC}\cdot(B_1 - P_{CC})] + [t(P_{CC} + h\beta P_{LL})\cdot(B_2 + gP_{CC} - P_{CC} - \beta P_{LL})] + [(1-t)(1-h)\alpha P_{CC}(A_2 + f P_{LL} - P_{LL} - \alpha P_{CC})] - [C_{CC}\cdot(B_1 - P_{CC}) + (1+\delta)C_{CC}\cdot(B_2 + g P_{CC} - P_{CC} - \beta P_{LL})]$$

where the first bracket of (9) represents revenues of the landline phone company from calls to their landline customers. The second bracket represents the landline company revenue from calls of landline customers to cellular customers. The third bracket represents the revenues delivered by the cellular companies to the landline customers. The same three brackets of (10) represent revenues earned by the cellular company. The last bracket in both equations represents the total cost of direct and cross-calls between companies.

Both companies maximize their profit with respect to its decision variables, P_{LL}, the decision variable of the landline company and P_{CC}, the decision variable of the landline cellular company.

We derivate as follows[2]:

(11)

$$\frac{\partial \Pi_L}{\partial P_{LL}} = t(A_1 - P_{LL}) - tP_{LL} + (1-t)\cdot(A_2 + fP_{LL} - P_{LL} - \alpha P_{CC}) - (1-t)(1-f)(P_{LL} + h\alpha P_{CC}) + t(1-h)\beta(B_2 + gP_{CC} - P_{CC} - \beta P_{LL}) - t(1-h)\beta^2 P_{LL} + C_{LL} + (1+\gamma)(1-f)C_{LL} = 0$$

(12)

$$\frac{\partial \Pi_C}{\partial P_{CC}} = B_1 - P_{CC} - P_{CC} + t[B_2 - (1-g)P_{CC} - \beta P_{LL}] - t(1-g)(P_{CC} + h\beta P_{LL}) + (1-t)(1-h)\alpha[A_2 - (1-f)P_{LL} - \alpha P_{CC}] - (1-t)(1-h)\alpha^2 P_{CC} + C_{CC} + (1+\delta)(1-g)C_{CC} = 0$$

From equation (11) we can derive the reaction function of the landline phone company's pricing policy, P_{LL}, as a function of the cellular company's price, P_{CC}, i.e.,

(13)

$$P_{LL} = \frac{\{tA_1 + (1-t)A_2 + t(1-h)\beta B_2 + [1 + (1+\gamma)(1-f)C_{LL}]\}}{[2t + 2(1-t)(1-f) + 2t(1-h)\beta^2]} - \frac{\{(1-t)\alpha + [(1-t)(1-f)h\alpha + t(1-h)(1-g)\beta]\}}{[2t + 2(1-t)(1-f) + 2t(1-h)\beta^2]} P_{CC}$$

From equation (12) we can repeat the same procedure and find the reaction function of the cellular company pricing policy, P_{CC}, as a function of the landline company price, P_{LL}, that is:

(14)

$$P_{CC} = \frac{\{B_1 + tB_2 + (1-t)(1-h)\alpha A_2 + [1 + (1+\delta)(1-g)C_{CC}]\}}{[2 + 2t(1-g) + 2(1-t)(1-h)\alpha^2]} - \frac{\{[t\beta + t\beta h(1-g) + (1-t)(1-h)(1-f)\alpha]\}}{[2 + 2t(1-g) + 2(1-t)(1-h)\alpha^2]} P_{LL}$$

where $X = [2t + 2(1-t)(1-f) + 2t(1-h)\beta^2]$

Defining the denominator of (13) as *X* where:

$$[2t + 2(1-t)(1-f) + 2t(1-h)\beta^2]$$

The numerator of the first term of (13) as *U* where:

$$U = \{tA_1 + (1-t)A_2 + t(1-h)\beta B_2 + [1 + (1+\gamma)(1-f)C_{LL}]\}$$

and the second term numerator of (13) as *W*, where:

$$W = \{(1-t)\alpha + [(1-t)(1-f)h\alpha + t(1-h)(1-g)\beta]\}$$

While the denominator of (14) we define as *Y* where:

$$Y = [2 + 2t(1-g) + 2(1-t)(1-h)\alpha^2]$$

The first term numerator of (14) as *V* where:

$$V = \{B_1 + tB_2 + (1-t)(1-h)\alpha A_2 + [1 + (1+\delta)(1-g)C_{CC}]\}$$

and the second term numerator as *Z* where:

$$Z = \{[t\beta + t\beta h(1-g) + (1-t)(1-h)(1-f)\alpha]\}$$

[2] S.O.C. were examined and can be introduced upon request.

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We can rewrite equations (13) and (14) as follows:

$$(13')\quad P_{LL} + \left(\frac{W}{X}\right)P_{CC} = \frac{U}{X}$$

and

$$(14')\quad \left(\frac{Z}{Y}\right)P_{LL} + P_{CC} = \frac{V}{Y}$$

The two reaction functions can also be written as follows:

The reaction function of the landline company as

$$(13'')\quad P_{LL} = \left(\frac{U}{X}\right) - \left(\frac{W}{X}\right)P_{CC}$$

and the inverse reaction function of the cellular company as

(14'') $P_{LL} = \frac{V}{Z} - \left(\frac{Y}{Z}\right)P_{CC}$ that we demonstrate graphically in Figure 1.

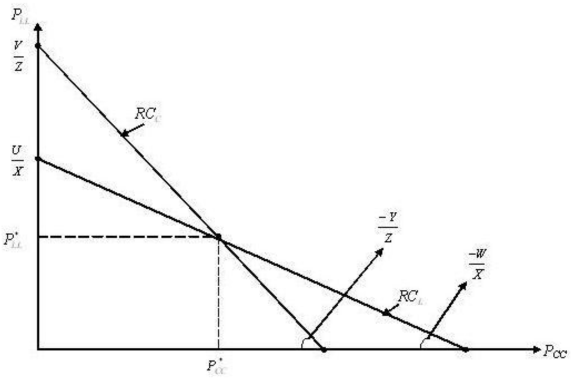

Figure 1

For a stable equilibrium, P^*_{LL} and P^*_{CC}, the following two conditions are required:

1. The linear RC_L illustrated in equation (13'') should be flatter than the RC_C of equation (14'').
2. The intercept of RC_L on the Y axis should be smaller than the intercept of RC_C.[3]

These conditions hold if $\frac{W}{X} < \frac{Y}{Z}$ and $\frac{U}{X} < \frac{V}{Z}$

or

I. $XY > WZ$ and
II. $VX > UZ$

[3] Otherwise either no intersection point exists or it exists but is not a stable equilibrium.

The intersection point of the two linear functions shown on Figure 1 represents the Nash long-run equilibrium under non-cooperative game can be solved by Cramer system as follows:

(15)

$$P^*_{LL} = \frac{\begin{vmatrix} \frac{U}{X} & \frac{W}{X} \\ \frac{V}{Y} & 1 \end{vmatrix}}{\begin{vmatrix} 1 & \frac{W}{X} \\ \frac{Z}{Y} & 1 \end{vmatrix}} = \frac{\frac{U}{X} - \frac{WV}{XY}}{1 - \frac{WZ}{XY}} = \frac{UY - WV}{XY - WZ}$$

(16)

$$P^*_{CC} = \frac{\begin{vmatrix} 1 & \frac{U}{X} \\ \frac{Z}{Y} & \frac{V}{Y} \end{vmatrix}}{\begin{vmatrix} 1 & \frac{W}{X} \\ \frac{Z}{Y} & 1 \end{vmatrix}} = \frac{\frac{V}{Y} - \frac{UZ}{XY}}{1 - \frac{WZ}{XY}} = \frac{VX - UZ}{XY - WZ}$$

From equations (5), (6) and (15) and (16) we can determine the equilibrium values of P^*_{LC} and P^*_{CL} as follows:

$$(17)\ P^*_{LC} = P^*_{LL} + \alpha P^*_{CC} = \frac{UY - WV + \alpha(VX - UZ)}{XY - WZ}$$

$$(18)\ P^*_{CL} = P^*_{CC} + \beta P^*_{LL} = \frac{VX - UZ + \beta(UY - WV)}{XY - WZ}$$

From equation (1) – (4) and (15) – (18) we can determine the equilibrium values of $q^*_{LL}, q^*_{LC}, q^*_{CC}$ and q^*_{CL} as follows:

$$(19)\quad q^*_{LL} = A_1 - \left(\frac{UY - WV}{XY - WZ}\right)$$

(20)

$$q^*_{LC} = A_2 - \left[\frac{UY - WV + \alpha(VX - UZ)}{XY - WZ}\right] + f\left[\frac{UY - WZ}{XY - WZ}\right]$$

$$(21)\quad q^*_{CC} = \beta_1 - \left[\frac{VX - UZ}{XY - WZ}\right]$$

(22)

$$q^*_{CL} = \beta_2 - \left[\frac{VX - UZ + \beta(UY - WV)}{XY - WZ}\right] + G\left[\frac{VX - UX}{XY - WZ}\right]$$

From the equilibrium prices and quantities we can determine the appropriate values of *TR*, *TC* and Π of both companies.

3. COMPARATIVE STATIC ANALYSIS

In this section we investigate the effects of several controls exogenous variables on the price policy of the two phone companies: landline and cellular.

The first variable is α that represent the price share that the cellular company imposes on the call that is initiated from a regular phone to a receiver who receives the call by using a cellular phone/mobile phone. The question is how a change in α that can theoretically approach to a value of 1 will affect P_{LL} and P_{CC} and as a result also will affect the cross calls price from landline to cellular and vice versa.

Taking the derivative of (16) with respect to α yields the following:

(23)

$$\frac{\partial P_{CC}^{*}}{\partial \alpha}=\frac{\left(\frac{\partial V}{\partial \alpha}X+\frac{\partial X}{\partial \alpha}V-\frac{\partial U}{\partial \alpha}Z-\frac{\partial Z}{\partial \alpha}U\right)(XY-WZ)-\left(\frac{\partial X}{\partial \alpha}Y+\frac{\partial Y}{\partial \alpha}X-\frac{\partial W}{\partial \alpha}Z-\frac{\partial Z}{\partial \alpha}W\right)(VX-UZ)}{(XY-WZ)^{2}}$$

Since

III.

$$\frac{\partial X}{\partial \alpha}=\frac{\partial U}{\partial \alpha}=0;\quad \frac{\partial W}{\partial \alpha}=(1-t)[1+(1-f)h]>0,\quad \frac{\partial Y}{\partial \alpha}=4(1-t)(1-k)\alpha>0,$$

$$\frac{\partial Z}{\partial \alpha}=(1-t)(1-h)(1-f)>0,\quad \frac{\partial V}{\partial \alpha}=(1-t)(1-h)A_{2}>0$$

We can rewrite (23) as (23')

(23')

$$\frac{\partial P_{CC}^{*}}{\partial \alpha}=\frac{\frac{\partial V}{\partial \alpha}X(XY-WZ)-\frac{\partial Z}{\partial \alpha}X(UY-VW)+(UZ-VX)\left(\frac{\partial Y}{\partial \alpha}X-\frac{\partial W}{\partial \alpha}Z\right)}{(XY-WZ)^{2}}$$

Based on I, II, III above we conclude that the first right term of (23') is positive but relatively very small, since most likely the values are smaller than 1 and those values are multiplied by A_2. The summation of the other two values is negative and they are relatively larger in absolute values than the first right term. Thus, we can conclude that presumably $\frac{\partial P_{CC}}{\partial \alpha}<0$ although under very limited condition it is possible that a direct effect of parameter α on P_{CC}^{*}, i.e., a positive relationship.

The next consideration is presented at (24) where we examine the effect of α on P_{LL}^{*},

(24)

$$\frac{\partial P_{LL}^{*}}{\partial \alpha}=\frac{\left(\frac{\partial Y}{\partial \alpha}U+\frac{\partial U}{\partial \alpha}Y-\frac{\partial W}{\partial \alpha}V-\frac{\partial V}{\partial \alpha}W\right)(XY-WZ)-\left(\frac{\partial Y}{\partial \alpha}X+\frac{\partial X}{\partial \alpha}Y-\frac{\partial W}{\partial \alpha}Z-\frac{\partial Z}{\partial \alpha}W\right)(UY-WV)}{(XY-WZ)^{2}}$$

Using the above information we can rewrite (24) as (24')

(24')

$$\frac{\partial P_{LL}^{*}}{\partial \alpha}=\frac{\left[\frac{\partial W}{\partial \alpha}Y-\frac{\partial Y}{\partial \alpha}W\right](UZ-VX)-\frac{\partial V}{\partial \alpha}W(XY-WZ)+\frac{\partial Z}{\partial \alpha}W(UY-WV)}{(XY-WZ)^{2}}$$

Again we can conclude by using the conditions I – III above that the right side second term of the numerator is negative and is relatively small, while it is multiplied by A_1. The third term is positive and large. However, the first term value is definitely negative since the value in the bracket is as follows:

$$\frac{\partial W}{\partial \alpha}Y-\frac{\partial Y}{\partial \alpha}W=$$
$$(1-t)[1+(1-f)h]\left[2+2t(1-g)+2(1-t)(1-h)\alpha^{2}\right]-$$
$$-h(1-t)(1-h)\alpha[(1-t)\alpha+(1-t)(1-f)h\alpha+t(1-h)(1-g)\beta]=$$
$$(1-t)h\{(1-2f)\left[1+t(1-g)+(1-t)(1-h)\alpha^{2}\right]-\left[\alpha^{2}(1-t)(2-f)+\beta\alpha t(1-h)(1-g)\right]\}$$
$$>0$$

Since $(UZ-VX)$ is negative while the bracket is positive, the first right-hand term is negative as well as the second term. As such small negative values of these two terms cannot be compared to the third term that is positive. If it is a large positive value, i.e., $A_i >> B_i$ then $\frac{\partial P_{LL}}{\partial \alpha}>0$, otherwise it can be negative or zero.

A similar analysis, since the demand function for landline and cellular are symmetric, we can conclude that change in parameter β representing the payment the cellular company delivers to the landline company for cross calls from a cellular to a regular phone is again not clear:

$\frac{\partial P_{CC}^{*}}{\partial \beta}$ can be either positive or negative, still it is more likely to be negative, however $\frac{\partial P_{LL}^{*}}{\partial \beta}$ is most likely negative. Still there is very slim possibility that the sign will be positive.

The next stage is to analyze the effects of changes in parameters *g* and/or *f*, namely how an increase in the substitution degree of the two goods will affect the pricing policy by of two goods determined the two companies.

Taking the derivative of (16) with respect of g yields:

(25)

$$\frac{\partial P^*_{CC}}{\partial g}=\frac{\left[\frac{\partial V}{\partial g}X+\frac{\partial X}{\partial g}V-\frac{\partial U}{\partial g}Z-\frac{\partial Z}{\partial g}U\right](XY-WZ)-\left[\frac{\partial X}{\partial g}Y+\frac{\partial Y}{\partial g}X-\frac{\partial W}{\partial g}Z-\frac{\partial Z}{\partial g}W\right](VX-UZ)}{(XY-WZ)^2}$$

Since

IV. $\frac{\partial X}{\partial g}=\frac{\partial U}{\partial g}=0$ $\quad \frac{\partial Y}{\partial g}=-2t<0$ $\quad \frac{\partial Z}{\partial g}=-t\beta h<0$

$\frac{\partial V}{\partial g}=-(1+\delta)C_{CC}<0$ and $\frac{\partial W}{\partial g}=-\beta<0$,

We can rewrite 25 as (25')

(25')

$$\frac{\partial P^*_{CC}}{\partial g}=\frac{\frac{\partial V}{\partial g}X(XY-WZ)-\frac{\partial Z}{\partial g}X(UY-VW)+\left(\frac{\partial W}{\partial g}Z-\frac{\partial Y}{\partial g}X\right)(VX-UZ)}{(XY-WZ)^2}$$

The first term of the right hand is negative but relatively small, while the other two terms are positive in sign and significantly larger. Thus we can conclude from (25') that $\frac{\partial P^*_{CC}}{\partial g}>0$. From the derivative of (15) with respect to g we get:

(26)

$$\frac{\partial P^*_{LL}}{\partial g}=\frac{\left(\frac{\partial Y}{\partial g}U+\frac{\partial U}{\partial g}Y-\frac{\partial W}{\partial g}V-\frac{\partial V}{\partial g}W\right)(XY-WZ)-\left(\frac{\partial Y}{\partial g}X+\frac{\partial X}{\partial g}Y-\frac{\partial W}{\partial g}Z-\frac{\partial Z}{\partial g}W\right)(UY-VW)}{(XY-WZ)^2}$$

Using condition I, II, and IV above we can rewrite (26) as (26')

(26')

$$\frac{\partial P^*_{LL}}{\partial g}=\frac{\left(\frac{\partial Y}{\partial g}W-\frac{\partial W}{\partial g}Y\right)(VX-UZ)+\frac{\partial V}{\partial g}W(WZ-XY)+\frac{\partial Z}{\partial g}W(UY-VW)}{(XY-WZ)^2}$$

The left term of the numerator is ambiguous in sign since $\frac{\partial Y}{\partial g}<0$ while $\frac{\partial W}{\partial g}<0$ but it is hard to determine which of the two is larger. The second term is positive while the third term of the numerator is negative. Therefore, the sign of $\frac{\partial P^*_{LL}}{\partial g}$ is ambiguous. The conclusion can be different for various values of the parameters. For extreme values of *t* (either approaching to value one or zero) as well as extreme values of parameters h,α,β and significant gap between *g* and *f* values indeed the sign is ambiguous. However, for average values of the parameters above it is more likely that the sign of $\frac{\partial P^*_{LL}}{\partial g}$ is positive.

Let us now discuss the effect of parameter f on optimal values of P^*_{CC} and P^*_{LL} as follows:

By taking the derivative of (16) with respect to f we get:

(27)

$$\frac{\partial P^*_{CC}}{\partial f}=\frac{\left[\frac{\partial V}{\partial f}X+\frac{\partial X}{\partial f}V-\frac{\partial U}{\partial f}Z-\frac{\partial Z}{\partial f}U\right](XY-WZ)-\left[\frac{\partial X}{\partial f}Y+\frac{\partial Y}{\partial f}X-\frac{\partial W}{\partial f}Z-\frac{\partial Z}{\partial f}W\right](VX-UZ)}{(XY-WZ)^2}$$

Since

V. $\frac{\partial V}{\partial f}=\frac{\partial Y}{\partial f}=0$, $\quad \frac{\partial W}{\partial f}=-(1-t)h\alpha<0$,

$\frac{\partial X}{\partial f}=-2(1-t)<0$, $\frac{\partial Z}{\partial f}=-(1-t)(1-h)\alpha<0$

We can rewrite (27) as (27')

(27') $$\frac{\partial P^*_{CC}}{\partial f}=\frac{-\frac{\partial U}{\partial f}Z(XY-WZ)-\frac{\partial W}{\partial f}Z(VX-UZ)+\left(\frac{\partial X}{\partial f}Z-\frac{\partial Z}{\partial f}X\right)(UY-WV)}{(XY-WZ)^2}$$

Based on I, II, III and IV we can conclude that the two left terms of the numerator as well as the denominator are all positive. However, the third term of the numerator is negative. The arguments discussed above regarding the value $\frac{\partial P^*_{LL}}{\partial f}$ are the same as $\frac{\partial P^*_{CC}}{\partial f}$ with the same conclusion where in extreme cases of the parameters the sign is not clear. However, for the intermediate case, it is more likely that the sign is positive.

From the derivative of (15) with respect to f we get:

(28)

$$\frac{\partial P^*_{LL}}{\partial f}=\frac{\left(\frac{\partial U}{\partial f}Y+\frac{\partial Y}{\partial f}U-\frac{\partial W}{\partial f}V-\frac{\partial V}{\partial f}W\right)(XY-WZ)-\left(\frac{\partial X}{\partial f}Y+\frac{\partial Y}{\partial f}X-\frac{\partial W}{\partial f}Z-\frac{\partial Z}{\partial f}W\right)(UY-WV)}{(XY-WZ)^2}$$

Based on I, II, III and V above we can rewrite (28) as (28"):

(28')

$$\frac{\partial P^*_{LL}}{\partial f}=\frac{\frac{\partial U}{\partial f}Y(XY-WZ)-\frac{\partial W}{\partial f}Y(VX-UZ)-\left(\frac{\partial X}{\partial f}Y-\frac{\partial Z}{\partial f}W\right)(UY-WV)}{(XY-WZ)^2}$$

In this case the first term of the numerator is negative. However, the other two terms are positive and large therefore we can conclude that most likely the value is positive: $\frac{\partial P^*_{LL}}{\partial f}>0$.

In the next stage we examine the effect of the value t on P^*_{LL} and P^*_{CC}. By taking the derivatives of (15) and (16) with respect to t change we get:

(29)

$$\frac{\partial P_{LL}^{*}}{\partial t}=\frac{\left(\frac{\partial U}{\partial t}Y+\frac{\partial U}{\partial f}U-\frac{\partial W}{\partial t}V-\frac{\partial V}{\partial t}W\right)(XY-WZ)-\left(\frac{\partial X}{\partial t}Y+\frac{\partial Y}{\partial f}X+\frac{\partial W}{\partial t}Z-\frac{\partial Z}{\partial f}W\right)(UY-WV)}{(XY-WZ)^{2}}$$

Where

VI. $\frac{\partial X}{\partial t}=2\left[(1-h)\beta^{2}+f\right]>0$,

$\frac{\partial W}{\partial t}=(1-h)(1-g)\beta-\alpha\left[1+h(1-f)\right]$,

i.e., its sign is not clear

$\frac{\partial U}{\partial t}=A_{1}-A_{2}+(1-h)\beta B_{2}>0$,

$\frac{\partial Y}{\partial t}=2\left[1-g-(1-h)\alpha^{2}\right]>0$,

$\frac{\partial Z}{\partial t}=\beta\left[1-h(1-g)\right]-(1-h)(1-f)\alpha$ i.e., its sign is not clear,

$\frac{\partial V}{\partial t}=\beta_{2}-(1-h)\alpha A$,

i.e., again its sign is not clear

From (29) and conditions VI we can rewrite (29) as (29')

(29')

$$\frac{\partial P_{LL}^{*}}{\partial t}=\frac{\left(\frac{\partial U}{\partial t}Y-\frac{\partial V}{\partial t}W\right)(XY-WZ)-\left(\frac{\partial X}{\partial t}Y-\frac{\partial Z}{\partial t}W\right)(UY-WV)+\left(\frac{\partial Y}{\partial t}W-\frac{\partial W}{\partial t}Y\right)(VX-UZ)}{(XY-WZ)^{2}}$$

All three terms of the numerator are not defined in sign terms therefore the value of the derivative of (29') is not clear. This indicates that an increase in availability of a call receiver on a regular phone does not guarantee an increase in demand, thus in price of phone calls from landline to landline.

By the same procedure we take the derivative of P_{cc} with respect to change in parameter *t*

(30)

$$\frac{\partial P_{CC}^{*}}{\partial t}=\frac{\left(\frac{\partial V}{\partial t}X+\frac{\partial X}{\partial t}V-\frac{\partial U}{\partial t}Z-\frac{\partial Z}{\partial t}U\right)(XY-WZ)-\left(\frac{\partial X}{\partial t}Y+\frac{\partial Y}{\partial f}X-\frac{\partial W}{\partial t}Z-\frac{\partial Z}{\partial f}W\right)(VX-UZ)}{(XY-WZ)^{2}}$$

From (30) and VI above we get (30')

(30')

$$\frac{\partial P_{CC}^{*}}{\partial t}=\frac{\left(\frac{\partial V}{\partial t}X-\frac{\partial U}{\partial t}Z\right)(XY-WZ)+\left(\frac{\partial W}{\partial t}Z-\frac{\partial Y}{\partial t}X\right)(VX-UZ)+\left(\frac{\partial X}{\partial t}Z-\frac{\partial Z}{\partial t}X\right)(UY-WV)}{(XY-WZ)^{2}}$$

Again the effect of increase in parameter *t* on the substitute tools (cellular) price is not clear.

The next investigation deals with changes in parameter *h*, i.e., the cross sharing revenues between "cooperative rivals": the supervision of calls from the landline company and the cellular company. The question we ask is how an increase in the revenues that transferred from cross calls between the cellular and regular phone and vice versa could affect the pricing policies for both companies.

Again we take the derivative of (15) and (16) with respect to *h* as follows:

(31)

$$\frac{\partial P_{LL}^{*}}{\partial h}=\frac{\left(\frac{\partial U}{\partial h}Y+\frac{\partial Y}{\partial h}U-\frac{\partial W}{\partial h}V-\frac{\partial V}{\partial h}W\right)(XY-WZ)-\left(\frac{\partial X}{\partial h}Y+\frac{\partial Y}{\partial h}X-\frac{\partial W}{\partial h}Z-\frac{\partial Z}{\partial h}W\right)(VX-UZ)}{(XY-WZ)^{2}}$$

Since

VII. $\frac{\partial X}{\partial h}=2t\beta^{2}<0$,

$\frac{\partial W}{\partial h}=(1-t)(1-f)\alpha-t(1-g)\beta$ i.e., its sign is not clear,

$\frac{\partial U}{\partial h}=t\beta B_{2}<0$, $\frac{\partial Y}{\partial h}=-2(1-t)\alpha^{2}<0$,

$\frac{\partial Z}{\partial h}=-(1-t)(1-f)\alpha+t\beta(1-g)$ i.e., its sign is again not clear, $\frac{\partial V}{\partial h}=-(1-t)\alpha A_{2}<0$

We can rewrite (31) as (31')

(31')

$$\frac{\partial P_{LL}^{*}}{\partial h}=\frac{\left(\frac{\partial U}{\partial h}Y-\frac{\partial V}{\partial h}W\right)(XY-WZ)+\left(\frac{\partial Z}{\partial h}W-\frac{\partial X}{\partial h}Y\right)(UY-VW)+\left(\frac{\partial Y}{\partial h}W-\frac{\partial W}{\partial h}Y\right)(VX-UZ)}{(XY-WZ)^{2}}$$

The first two terms of the numerator are positive while the third term is not defined. However the positive second value is larger in absolute term than the third term, therefore it is almost definitely clear that an increase in sharing will lead to an increase in the value P_{LL}^{*}.

The derivative of P_{CC} with respect to *h* is presented below at (32)

(32)

$$\frac{\partial P_{CC}^{*}}{\partial h}=\frac{\left(\frac{\partial V}{\partial h}X+\frac{\partial X}{\partial h}V-\frac{\partial U}{\partial h}Z-\frac{\partial Z}{\partial h}U\right)(XY-WZ)-\left(\frac{\partial X}{\partial h}Y+\frac{\partial Y}{\partial h}X-\frac{\partial W}{\partial h}Z-\frac{\partial Z}{\partial h}W\right)(VX-UZ)}{(XY-WZ)^{2}}$$

From (32) and VII above we can rewrite (32) as (32')

(32')

$$\frac{\partial P_{CC}^{*}}{\partial h}=\frac{\left(\frac{\partial V}{\partial h}X-\frac{\partial U}{\partial h}Z\right)(XY-WZ)+\left(\frac{\partial W}{\partial h}Z-\frac{\partial Y}{\partial h}X\right)(VX-UZ)+\left(\frac{\partial X}{\partial h}Z-\frac{\partial Z}{\partial h}X\right)(UY-VW)}{(XY-WZ)^{2}}$$

The first two terms of the numerator are negative, while the sign of the third term is not clear. By investigating the specific values of the third term, we can conclude that presumably the term is negative, but even if it is positive the absolute value of this term is small and most likely smaller than the other two large negative terms. Therefore, it is most likely that the sign of the term $\frac{\partial P_{CC}^*}{\partial h}$ is negative.

The last two parameters γ and δ influence the production cost functions. As can be expected the increase in the direct costs on price are positive. However, the cross effects are negative namely the increase in the cost of production one item may lead to a decrease in the price of the partial substitute good. The formal proofs are discussed below:

$$(33)\quad \frac{\partial P_{LL}^*}{\partial \gamma} = \frac{\frac{\partial U}{\partial \gamma} Y \cdot (XY - WZ)}{(XY - WZ)^2} > 0$$

Since:

VIII. $\frac{\partial X}{\partial \gamma} = \frac{\partial W}{\partial \gamma} = \frac{\partial Y}{\partial \gamma} = \frac{\partial Z}{\partial \gamma} = \frac{\partial V}{\partial \gamma} = 0$ and $\frac{\partial U}{\partial \gamma} = (1 - f) C_{LL} > 0$

$$(34)\quad \frac{\partial P_{CC}^*}{\partial \gamma} = \frac{\frac{\partial U}{\partial \gamma} Z(XY - WZ)}{(XY - WZ)^2} < 0$$

The proof of the change in δ is similar:

$$(35)\quad \frac{\partial P_{LL}^*}{\partial \delta} = \frac{-\frac{\partial V}{\partial \delta} W(XY - WZ)}{(XY - WZ)^2} < 0$$

Since

IX. $\frac{\partial X}{\partial \delta} = \frac{\partial W}{\partial \delta} = \frac{\partial U}{\partial \delta} = \frac{\partial Y}{\partial \delta} = \frac{\partial Z}{\partial \delta} = 0$, but $\frac{\partial U}{\partial \delta} = (1 - g) C_{LL} > 0$

However, the effect of δ on P_{CC}^* is derived as follows:

$$(36)\quad \frac{\partial P_{CC}^*}{\partial \delta} = \frac{-\frac{\partial V}{\partial \delta} X(XY - WZ)}{(XY - WZ)^2} > 0$$

Numerical example

We now demonstrate a simple example for our model to simplify it.

The numbers we choose are based on all the assumptions mentioned in previous section:
A_1=67, A_2=55, B_1=50, B_2=49, C_{LL}=32, C_{CC}=25, α =0.6, β=0.5, f=0.5, g=0.6, γ =0.3, δ=0.4, t=0.6 and h=0.6

Therefore the basic equations can be rewritten as follows:

$$(37)\quad q_{LL} = A_1 - P_{LL} = 67 - P_{LL}$$

$$(38)\quad q_{LC} = A_2 - P_{LC} + fP = 55 - P_{LC} + 0.5P_{LL}$$

$$(39)\quad q_{CC} = B_1 - P_{CC} = 50 - P_{CC}$$

$$(40\quad q_{CL} = B_2 - P_{CL} + gP_{CC} = 49 - P_{CL} + 0.6P_{CC}$$

$$(41)\quad P_{LC} = P_{LL} + \alpha P_{CC} = P_{LL} + 0.6 P_{CC}$$

$$(42)\quad P_{CL} = P_{CC} + \beta P_{LL} = P_{CC} + 0.5 P_{LL}$$

$$(43)\quad C_{LC} = (1+\gamma)\, C_{LL} = (1+0.3)\, 32 = 41.6$$

$$(44)\quad C_{CL} = (1+\delta)\, C_{CC} = (1+0.4)\, 25 = 35$$

The profits of the companies are:

(45)

$$\pi_L = [t P_{LL}(A_1 - P_{LL})] + [(1-t)(P_{LL} + h\alpha P_{CC}) \cdot (A_2 + f \cdot P_{LL} - P_{LL-} - \alpha P_{CC})] + $$
$$+ [t(1-h)\beta P_{LL}(B_2 + gP_{CC} - P_{CC} - \beta P_{LL})] -$$
$$- [C_{LL} \cdot (A_1 - P_{LL}) + (1+\gamma) C_{LL} \cdot (A_2 + f P_{LL} - P_{LL} - \alpha P_{CC})] =$$

$$= 0.6 P_{LL}(67 - P_{LL}) + 0.4(P_{LL} + 0.6 \cdot 0.6 P_{CC}) \cdot (55 + 0.5 \cdot P_{LL} - P_{LL-} - 0.6P_{CC}) +$$
$$+ 0.6 \cdot 0.4 \cdot 0.5 P_{LL}(49 + 0.6P_{CC} - P_{CC} - 0.5 P_{LL}) -$$
$$- 32 \cdot (67 - P_{LL}) + (1 + 0.3) 32 \cdot (55 + 0.5 P_{LL} - P_{LL} - 0.6 P_{CC})$$

(46)

$$\pi_C = [P_{CC} \cdot (B_1 - P_{CC})] + [t(P_{CC} + h\beta P_{LL}) \cdot (B_2 + gP_{CC} - P_{CC} - \beta P_{LL})] +$$
$$+ [(1-t)(1-h)\alpha P_{CC}(A_2 + f P_{LL} - P_{LL} - \alpha P_{CC})] -$$
$$- [C_{CC} \cdot (B_1 - P_{CC}) + (1+\delta) C_{CC} \cdot (B_2 + g P_{CC} - P_{CC} - \beta P_{LL})]$$

$$P_{CC} \cdot (50 - P_{CC}) + 0.6 \cdot (P_{CC} + 0.6 \cdot 0.5 P_{LL}) \cdot (49 + 0.6P_{CC} - P_{CC} - 0.5 P_{LL}) +$$
$$+ 0.4 \cdot 0.4 \cdot 0.6 P_{CC}(55 + 0.5P_{LL} - P_{LL} - 0.6 P_{CC}) -$$
$$- 25 \cdot (50 - P_{CC}) + (1 + 0.4) 25 \cdot (49 + 0.6 P_{CC} - P_{CC} - 0.5 P_{LL})$$

The F.O.C. for maximization are:

(47)

$$\frac{\partial \pi_L}{\partial P_{LL}} = t(A_1 - P_{LL}) - tP_{LL} + (1-t) \cdot (A_2 + fP_{LL} - P_{LL} - \alpha P_{CC}) -$$
$$- (1-t)(1-f)(P_{LL} + h\alpha P_{CC}) + t(1-h)\beta(B_2 + gP_{CC} - P_{CC} - \beta P_{LL}) -$$
$$- t(1-h)\beta^2 P_{LL} + C_{LL} + (1+\gamma)(1-f) C_{LL} = 0$$

$$= 0.6 \cdot (67 - P_{LL}) - 0.6P_{LL} + 0.4 \cdot (55 + 0.5P_{LL} - P_{LL} - 0.6P_{CC}) -$$
$$- 0.4 \cdot 0.5 \cdot (P_{LL} + 0.6 \cdot 0.6P_{CC}) + 0.6 \cdot 0.4 \cdot 0.5 \cdot (49 + 0.6P_{CC} - P_{CC} - 0.5P_{LL}) -$$
$$- 0.6 \cdot 0.4 \cdot 0.5^2 P_{LL} + 32 + (1 + 0.3) 0.5 \cdot 32 = 0$$

(48)

$$\frac{\partial \pi_C}{\partial P_{CC}} = B_1 - P_{CC} - P_{CC} + t\left[B_2 - (1-g)P_{CC} - \beta P_{LL}\right] - t(1-g)(P_{CC} + h\beta P_{LL}) +$$
$$+ (1-t)(1-h)\alpha\left[A_2 - (1-f)P_{LL} - \alpha P_{CC}\right] - (1-t)(1-h)\alpha^2 P_{CC} +$$
$$+ C_{CC} + (1+\delta)(1-g)C_{CC} = 0$$

$$= 50 - P_{CC} - P_{CC} + 0.6\cdot\left[49 - 0.4P_{CC} - 0.5P_{LL}\right] - 0.6\cdot 0.4\cdot(P_{CC} + 0.6\cdot 0.5P_{LL}) +$$
$$+ 0.4\cdot 0.4\cdot 0.6\cdot\left[55 - 0.5P_{LL} - 0.6P_{CC}\right] - 0.4\cdot 0.4\cdot 0.6^2 P_{CC} +$$
$$+ 25 + (1+0.4)0.4\cdot 25 = 0$$

From the last two equations we can derive the reaction curves:

(49)

$$P_{LL} = \frac{\{tA_1 + (1-t)A_2 + t(1-h)\beta B_2 + [1 + (1+\gamma)(1-f)C_{LL}]\}}{\left[2t + 2(1-t)(1-f) + 2t(1-h)\beta^2\right]} -$$
$$- \frac{\{(1-t)\alpha + [(1-t)(1-f)h\alpha + t(1-h)(1-g)\beta]\}}{\left[2t + 2(1-t)(1-f) + 2t(1-h)\beta^2\right]} P_{CC} =$$
$$= \frac{\{0.6\cdot 67 + 0.4\cdot 55 + 0.6\cdot 0.4\cdot 0.5\cdot 49 + [1 + (1+0.3)\cdot 0.5\cdot 32]\}}{(2\cdot 0.6 + 2\cdot 0.4\cdot 0.5 + 2\cdot 0.6\cdot 0.4\cdot 0.5^2)} -$$
$$- \frac{\{0.6\cdot 0.6 + [0.4\cdot 0.5\cdot 0.6\cdot 0.6 + 0.6\cdot 0.4\cdot 0.4\cdot 0.5]\}}{(2\cdot 0.6 + 2\cdot 0.4\cdot 0.5 + 2\cdot 0.6\cdot 0.4\cdot 0.5^2)} P_{CC} =$$
$$= 70.3 - 0.21P_{CC}$$

(50)

$$P_{CC} = \frac{\{B_1 + tB_2 + (1-t)(1-h)\alpha A_2 + [1 + (1+\delta)(1-g)C_{CC}]\}}{\left[2 + 2t(1-g) + 2(1-t)(1-h)\alpha^2\right]} -$$
$$- \frac{\{[t\beta + t\beta h(1-g) + (1-t)(1-h)(1-f)\alpha]\}}{\left[2 + 2t(1-g) + 2(1-t)(1-h)\alpha^2\right]} P_{LL}$$
$$= \frac{\{50 + 0.6\cdot 49 + 0.4\cdot 0.4\cdot 0.6\cdot 55 + [1 + (1+0.4)0.4\cdot 25]\}}{(2 + 2\cdot 0.6\cdot 0.4 + 2\cdot 0.4\cdot 0.4\cdot 0.6^2)} -$$
$$- \frac{\{[0.6\cdot 0.5 + 0.6\cdot 0.5\cdot 0.6\cdot 0.4 + 0.4\cdot 0.4\cdot 0.5\cdot 0.6]\}}{(2 + 2\cdot 0.6\cdot 0.4 + 2\cdot 0.4\cdot 0.4\cdot 0.6^2)} P_{LL}$$
$$= 47.7 - 0.16P_{LL}$$

From (49) and (50) we get the prices of equilibrium determined by both companies:

P_{LL} =	62.42
P_{CC} =	37.56
P_{LC} =	84.95
P_{CL} =	68.76
q_{LL} =	4.58
Qcc =	12.44
q_{LC} =	1.23
q_{CL} =	2.77

At this stage we start a comparative static analysis. We demonstrate first the affect of α parameter changes:

	Basic State	**New State**
α =	0.6	0.8
PLL =	62.42	60.21
PCC =	37.56	36.13

From the simple example we can see that when α increases both prices at the new equilibrium are reduced.

Change in parameter β:

	Basic State	**New State**
β=	0.5	0.8
PLL=	62.42	58.32
PCC=	37.56	32.62

From the simple example we can see that when β increases both prices at the new equilibrium are reduced.

Change in parameter *g* may lead to two different possibilities:

	Possibility 1		*Possibility 2**	
	Basic State	**New State**	**Basic State**	**New State**
g=	0.6	0.8	0.6	0.8
h=	0.6	0.6	0.8	0.8
PLL=	62.42	62.59	62.96	62.77
PCC=	37.56	39.34	37.28	39.47

*In possibility 2 we change h for 0.8 instead of 0.6.

Possibility 1: (for *h*=0.6) indicates that if *g* increases both equilibrium prices in both market increase.
Possibility 2: (for *h*=0.8) indicates that if *g* increases then landline price decrease and cellular price increase.

Change in parameter *f* again may lead to different result:

	Possibility 1		*Possibility 2**	
	Basic State	**New State**	**Basic State**	**New State**
f=	0.5	0.8	0.5	0.8
h=	0.6	0.6	0.8	0.8
PLL =	62.42	65.14	62.96	66.37
PCC =	37.56	37.84	37.28	37.09

*In possibility 2 we change h for 0.8 instead of 0.6.

Possibility 1: (for *h*=0.6) indicates that if *f* increases both equilibrium prices in both market increase.
Possibility 2: (for *h*=0.8) indicates that if *g* increases then landline price increase and cellular price decrease.

Change in parameter *t* may lead to three different possibilities:

	Possibility 1		Possibility 2 *		Possibility 3**	
	Basic State	New State	Basic Sta	New State	Basic Stat	New State
t=	0.6	0.8	0.6	0.8	0.6	0.8
g=	0.6	0.6	0.8	0.8	0.6	0.6
f=	0.5	0.5	0.5	0.5	0.3	0.3
PLL=	62.42	59.75	62.59	60.05	61.00	61.28
PCC=	37.56	36.99	39.34	40.17	37.33	36.47

* In possibility 2 we change g for 0.8 instead of 0.6.
**In possibility 3 we change f for 0.3 instead of 0.5.

Possibility 1: (for *g*=0.6 and *f*=0.5) indicates that if *t* increases both equilibrium prices in both market decrease.
Possibility 2: (for *g*=0.8 and *f*=0.5) indicates that if *t* increases then landline price decrease and cellular price increase.
Possibility 3: (for *g*=0.6 and *f*=0.3) indicates that if *t* increases then landline price increase and cellular price decrease.
Change in parameter *h* may lead to three different possibilities:

	Possibility 1		Possibility 2*		Possibility 3**	
	Basic State	New State	Basic State	New State	Basic State	New State
h=	0.6	0.8	0.6	0.8	0.6	0.8
g=	0.6	0.6	0.8	0.8	0.6	0.6
t=	0.6	0.6	0.6	0.6	0.2	0.2
PLL=	62.42	62.96	62.59	62.77	70.17	68.66
PCC=	37.56	37.28	39.34	39.47	39.29	40.31

* In possibility 2 we change g for 0.8 instead of 0.6.
**In possibility 3 we change t for 0.2 instead of 0.6.

Possibility 1 : (for *g*=0.6 and *t*=0.6)) indicates that if *h* increases then landline price increase and cellular price decrease.
Possibility 2: (for *g*=0.8 and *t*=0.6) indicates that if *h* increases both equilibrium prices in both market increase.
Possibility 3: (for *g*=0.6 and *t*=0.2) indicates that if *h* increases then landline price decrease and cellular price increase.

Change in parameter γ:

	Basic State	New State
γ =	0.3	0.8
PLL =	62.42	67.23
PCC =	37.56	36.78

From the simple example we can see that if γ increase then landline price increase and cellular price decrease.

Change in parameter δ:

	Basic State	New State
δ =	0.4	0.8
PLL =	62.42	62.08
PCC =	37.56	39.15

From the simple example we can see that if δ increase then landline price decrease and cellular price increase.

4. IMPLICATIONS AND CONCLUSIONS

The model we developed in this paper has two important perspectives theoretical as well as practical.

From the theoretical point of view we develop a model of a market since we face a case of two different companies whose objectives are to market connection services; however, although the services somewhat overlap, they are certainly not complete substitutes. In some sense the services are substitutes but in another sense they can also be complements (e.g., a cross call from a regular land phone to a cell phone are complements when the party receiving the call is on the move but when the party receiving the call can be reached by a regular phone both connective devices are substitutes). Moreover, the model structure is based on a assumption of some degree of cooperation between the two rival companies. Since cross calls are accomplished by mutual attitude of companies to allow connections between landline and cellular phones the two companies share revenues from cross calls.

Their pricing strategies are based on both kinds of revenues; direct revenues from landline to landline or from cellular to cellular calls, plus shared revenues received by both companies from cross calls. The cross effects of the price of one item on the quantity demanded of the other depends on two factors. The first is the extra costs generated by cross-over calls, and the second is the shared revenues generated by those calls. These factors may lead to a different optimal pricing policy of coordination and cooperation between the two companies that substantially differ from the classical pricing policy of the well-known regular duopoly case. The results are summarized below based on the above mentioned factors as well as on a comparative static analysis.

The use of comparative static analysis have led to some straightforward results, however some other results are surprising, counter intuitive and in some sense even provocative. We can summarize the results as follows:

As expected, any increase in the cost of production of a good leads to an increase in its price. However, the result of an increase in the cost of production of a good leads to a definite reduction in the price of its partial substitute (since cellular and land line phones are not full substitutes).

A greater degree of cooperative behavior amongst rival companies who supply partial substitute services that are based on a cross sharing of revenues between companies most likely has opposite effects on the optimal pricing policy. If in our system the companies that are in charge of outgoing calls (either regular or cellular) receive a higher share of the income than those who are in charge of incoming calls (in the case of cross calls), then the landline company will raise prices while the cellular company's optimal price most likely will decline. This asymmetry in price changes is very surprising since we may have intuitively expected to see prices change in the same direction if not in size.

Regarding parameters *g* and *f* that represent the cross demand sensitivity of quality demanded for one good as a function of the price of the substitute good we can say that although both effects are symmetric, it is still worthwhile to investigate their effects on the optimal pricing of companies. We find that when the quantity of calls demanded from cellular to land line is positive in sign but larger in value than the prices from cellular to cellular, it is clear that the cellular supplier will raise the prices from cellular to cellular. However, the effect on the prices from "regular to regular" phones is ambiguous. Usually it may lead to the same increase in prices of phone to phone calls, however it might also be negative especially in cases where the availability of a regular phone receiver is very small or very large (t values are extremely close to zero or to one). This is (again) a very interesting result.

The fifth parameter we investigate is *t*, which measures the daily availability of the land line phone receiver. If *t* approaches one, we say that the regular phone is as efficient as the cellular in generating a connection between a sender and a receiver. The straightforward conclusion might be that the high value of *t* leads directly to an increase in the price of a call from line to line, while the opposite occurs to the price of calls from cellular to cellular as a result of diminishing demand of the former service. However, our general equilibrium model which takes into account the indirect effects of the different call services indicates that the effects of parameter t on the P_{CC} and P_{LL} value are ambiguous.

Our last analysis in the previous section deals with the combined effect of cross call prices (since these calls are charged by both land line and cellular companies). In the case that a call from a regular to a cellular is charged by the land line and a significant charge is also imposed by the cellular company, it is probable that it will lead to an increase in the total price of cross calls. However, in very rare cases the result can be the opposite i.e., a combined charge by both companies may lead to a reduction in the combined prices of the cross calls.

The comparison between solutions where only one firm offers one kind of service (either line or cellular) and the case where a multi-product firm or even several firms exist can be considered as important future extension to our current research.

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