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A Hybrid Method to Improve Forecasting Accuracy in the Case of Bread

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ABSTRACT

In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are many researches made on this. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method(ESM) is equivalent to (1,1) order ARMA model equation, new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the original production data of bread. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. The new method shows that it is useful for the time series that has various trend characteristics and has rather strong seasonal trend. The effectiveness of this method should be examined in various cases.

Keywords: *minimum variance, exponential smoothing method, forecasting, trend, bread*

1. INTRODUCTION

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM)[1]−[4]. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend[5], utilizing Kalman Filter[6], Bayes Forecasting[7], adaptive ESM[8], exponentially weighted Moving Averages with irregular updating periods [9], making averages of forecasts using plural method [10] are presented. For example, Maeda[6] calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise. It can be said that it does not pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii[11] pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn't show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before[12]. Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2*nd* order non-linear function and 3*rd* order non-linear function is executed to the original production data of bread. The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.

2. DESCRIPTION OF ESM USING ARMA MODEL[12]

In ESM, forecasting at time $t+1$ is stated in the following equation.

International Journal of Information and Communication Technology Research

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$$\hat{x}_{t+1} = \hat{x}_t + \alpha(x_t - \hat{x}_t) = \alpha x_t + (1-\alpha)\hat{x}_t \quad (1)$$

Here,

$\hat{x}_{t+1}$: forecasting at $t+1$
x_t : realized value at t
α : smoothing constant $(0 < \alpha < 1)$

(1) is re-stated as

$$\hat{x}_{t+1} = \sum_{l=0}^{\infty} \alpha(1-\alpha)^l x_{t-l} \quad (2)$$

By the way, we consider the following (1,1) order ARMA mod el.

$$x_t - x_{t-1} = e_t - \beta e_{t-1} \quad (3)$$

Generally, (p,q) order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j} \quad (4)$$

Here,

$\{x_t\}$:

Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1,2,\cdots,N,\cdots$

$\{e_t\}$: Gaussian White Noise with 0 mean σ_e^2 variance

MA process in (4) is supposed to satisfy convertibility conditio n. Utilizing the relation that

$$E[e_t | e_{t-1}, e_{t-2}, \cdots] = 0$$

We get the following equation from (3).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1} \quad (5)$$

Operating this scheme on $t+1$, we finally get

$$\hat{x}_{t+1} = \hat{x}_t + (1-\beta)e_t = \hat{x}_t + (1-\beta)(x_t - \hat{x}_t) \quad (6)$$

If we set $1-\beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to (1,1) order ARMA model, or is said to be (0,1,1) order ARIMA model because 1st order AR parameter is -1[1][3].

Comparing with (3) and (4), we obtain

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta \end{cases}$$

From (1), (6),

$$\alpha = 1 - \beta$$

Therefore, we get

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases} \quad (7)$$

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below.

Let (4) be

$$\tilde{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} \quad (8)$$

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j} \quad (9)$$

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (8), (9), we get the following non-linear equations which are well known[3].

$$\left.\begin{array}{l} \tilde{r}_k = \begin{cases} \sigma_e^2 \sum_{j=0}^{q-k} b_j b_{k+j} & (k \le q) \\ 0 & (k \ge q+1) \end{cases} \\ \tilde{r}_0 = \sigma_e^2 \sum_{j=0}^{q} b_j^2 \end{array}\right\} \quad (10)$$

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only b_1, so it can be solved in the following way.

From (3) (4) (7) (10), we get

$$\left.\begin{array}{l} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \\ \tilde{r}_0 = (1 + b_1^2)\sigma_e^2 \\ \tilde{r}_1 = b_1 \sigma_e^2 \end{array}\right\} \quad (11)$$

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0} \quad (12)$$

International Journal of Information and Communication Technology Research

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the following equation is derived.

$$\rho_1 = \frac{b_1}{1+b_1^2} \qquad (13)$$

We can get b_1 as follows.

$$b_1 = \frac{1 \pm \sqrt{1-4\rho_1^2}}{2\rho_1} \qquad (14)$$

In order to have real roots, ρ_1 must satisfy

$$|\rho_1| \le \frac{1}{2}$$

From invertibility condition, b_1 must satisfy

$$|b_1| < 1$$

From (13), using the next relation,

$$(1-b_1)^2 \ge 0$$
$$(1+b_1)^2 \ge 0$$

(15) always holds. As

$$\alpha = b_1 + 1$$

b_1 is within the range of

$$-1 < b_1 < 0$$

Finally we get

$$\left.\begin{aligned} b_1 &= \frac{1-\sqrt{1-4\rho_1^2}}{2\rho_1} \\ \alpha &= \frac{1+2\rho_1-\sqrt{1-4\rho_1^2}}{2\rho_1} \end{aligned}\right\} \qquad (16)$$

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

Here ρ_1 must satisfy

$$-\frac{1}{2} < \rho_1 < 0 \qquad (17)$$

in order to satisfy $0 < \alpha < 1$.

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter.

It can be estimated only by calculating 0th and 1st order autocorrelation function.

3. TREND REMOVAL METHOD[12]

As trend removal method, we describe the combination of linea r and non-linear function.

[1] Linear function
We set

$$y = a_1 x + b_1 \qquad (18)$$

as a linear function.

[2] Non-linear function
We set

$$y = a_2 x^2 + b_2 x + c_2 \qquad (19)$$

$$y = a_3 x^3 + b_3 x^2 + c_3 x + d_3 \qquad (20)$$

as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function
We set

$$y = \alpha_1(a_1 x + b_1) + \alpha_2(a_2 x^2 + b_2 x + c_2) \qquad (21)$$

$$y = \beta_1(a_1 x + b_1) + \beta_2(a_3 x^3 + b_3 x^2 + c_3 x + d_3) \qquad (22)$$

$$\begin{aligned} y &= \gamma_1(a_1 x + b_1) + \gamma_2(a_2 x^2 + b_2 x + c_2) \\ &\quad + \gamma_3(a_3 x^3 + b_3 x^2 + c_3 x + d_3) \end{aligned} \qquad (23)$$

as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Here, $\alpha_2 = 1-\alpha_1$, $\beta_2 = 1-\beta_1$, $\gamma_3 = 1-(\gamma_1+\gamma_2)$. Comparative discussion concerning (21), (22) and (23) are described in section 5.

4. MONTHLY RATIO[12]

For example, if there is the monthly data of L years as stated bellow:

$$\{x_{ij}\} \ (i = 1, \cdots, L) \ (j = 1, \cdots, 12)$$

Where, $x_{ij} \in R$ in which j means month and i means year and x_{ij} is a shipping data of i-th year, j-th month. Then, monthly ratio $\tilde{x}_j \ (j = 1, \cdots, 12)$ is calculated as follows.

$$\tilde{x}_j = \frac{\frac{1}{L}\sum_{i=1}^{L} x_{ij}}{\frac{1}{L}\cdot\frac{1}{12}\sum_{i=1}^{L}\sum_{j=1}^{12} x_{ij}} \qquad (24)$$

International Journal of Information and Communication Technology Research

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Monthly trend is removed by dividing the data by (24). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

5. FORECASTING THE SHIPPING DATA OF MANUFACTURER

5.1 Analysis Procedure

The production data of bread for 2cases from April 2007 to March 2010are analyzed. These data are obtained from the Annual Report of Statistical Investigation on Rice/Wheat Processed food by Ministry of Agriculture, Forestry and Fisheries in Japan. First of all, graphical charts of these time series data are exhibited in Figure 5-1,5-2. Here, Miscellaneous bread contains cooked bread (for example, salad is inserted), French roll, Hotel bread etc.

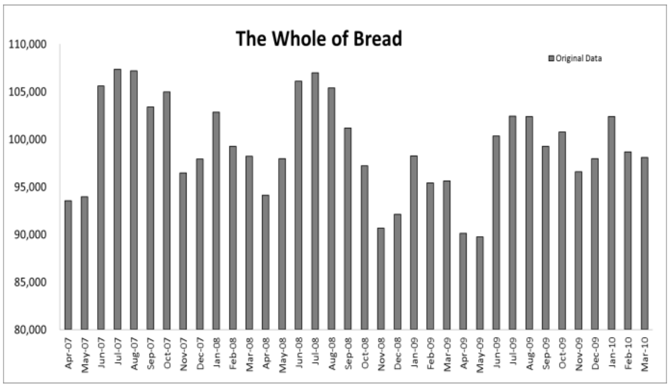

Figure 5-1: Production data of the Whole of Bread

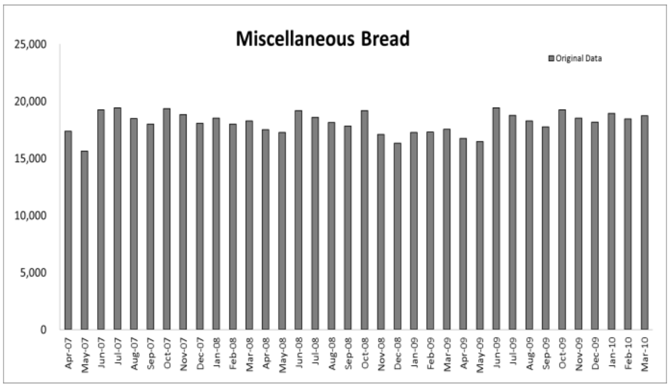

Figure 5-2: Production data of Miscellaneous Bread

Analysis procedure is as follows. There are 36 monthly data for each case. We use 24 data(1 to 24) and remove trend by the method stated in 3. Then we calculate monthly ratio by the method stated in 4. After removing monthly trend, the method stated in 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend.

Forecasting error is expressed as:

$$\varepsilon_i = \hat{x}_i - x_i \tag{25}$$

$$\bar{\varepsilon} = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i \tag{26}$$

Variance of forecasting error is calculated by:

$$\sigma_{\varepsilon}^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\varepsilon_i - \bar{\varepsilon}\right)^2 \tag{27}$$

5.2 Trend Removing

Trend is removed by dividing original data by,(21),(22),(23). The patterns of trend removal are exhibited in table5-1.

Table 5-1: The patterns of trend removal

Pattern1	α_1, α_2 are set 0.5 in the equation (21)
Pattern2	β_1, β_2 are set 0.5 in the equation (22)
Pattern3	α_1 is shifted by 0.01 increment in (21)
Pattern4	β_1 is shifted by 0.01 increment in (22)
Pattern5	γ_1 and γ_2 are shifted by 0.01 increment in (23)

In pattern1 and 2, the weight of α_1, α_2, β_1, β_2 are set 0.5 in the equation (21),(22). In pattern3, the weight of α_1 is shifted by 0.01 increment in (21) which satisfy the range $0 \le \alpha_1 \le 1.00$. In pattern4, the weight of β_1 is shifted in the same way which satisfy the range $0 \le \beta_1 \le 1.00$. In pattern5, the weight of γ_1 and γ_2 are shifted by 0.01 increment in (23) which satisfy the range $0 \le \gamma_1 \le 1.00$, $0 \le \gamma_2 \le 1.00$ .The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (18), (19) and (20) are exhibited in Table 5-2. Estimation results of weights of (21), (22) and (23) are exhibited in table 5-3

Volume 2 No. 11, November 2012 ISSN 2223-4985

International Journal of Information and Communication Technology Research

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Table 5-2 Coefficient of (18),(19) and (20)

	1st		2nd			3rd			
	a_1	b_1	a_2	b_2	c_2	a_3	b_3	c_3	d_3
The Whole of Bread	-233.535	102569.482	-37.831	712.227	98471.177	2.678	-138.240	1736.942	96121.590
Miscellaneous Bread	-32.920	18421.703	-8.503	179.657	17500.536	0.811	-38.908	489.952	16789.055

Table 5-3 weights of (21), (22) and (23)

	Monthly ratio	Pattern1		Pattern2		Pattern3		Pattern4		Pattern5		
		α_1	α_2	β_1	β_2	α_1	α_2	β_1	β_2	γ_1	γ_2	γ_3
The Whole of Bread	Used	0.5	0.5	0.5	0.5	1.00	0.00	1.00	0.00	1.00	0.00	0.00
	Not used	0.5	0.5	0.5	0.5	0.75	0.25	1.00	0.00	0.75	0.25	0.00
Miscellaneous Bread	Used	0.5	0.5	0.5	0.5	1.00	0.00	1.00	0.00	1.00	0.00	0.00
	Not used	0.5	0.5	0.5	0.5	1.00	0.00	1.00	0.00	1.00	0.00	0.00

Graphical chart of trend is exhibited in Figure 5-3, 5-4 for the cases that monthly ratio is used.

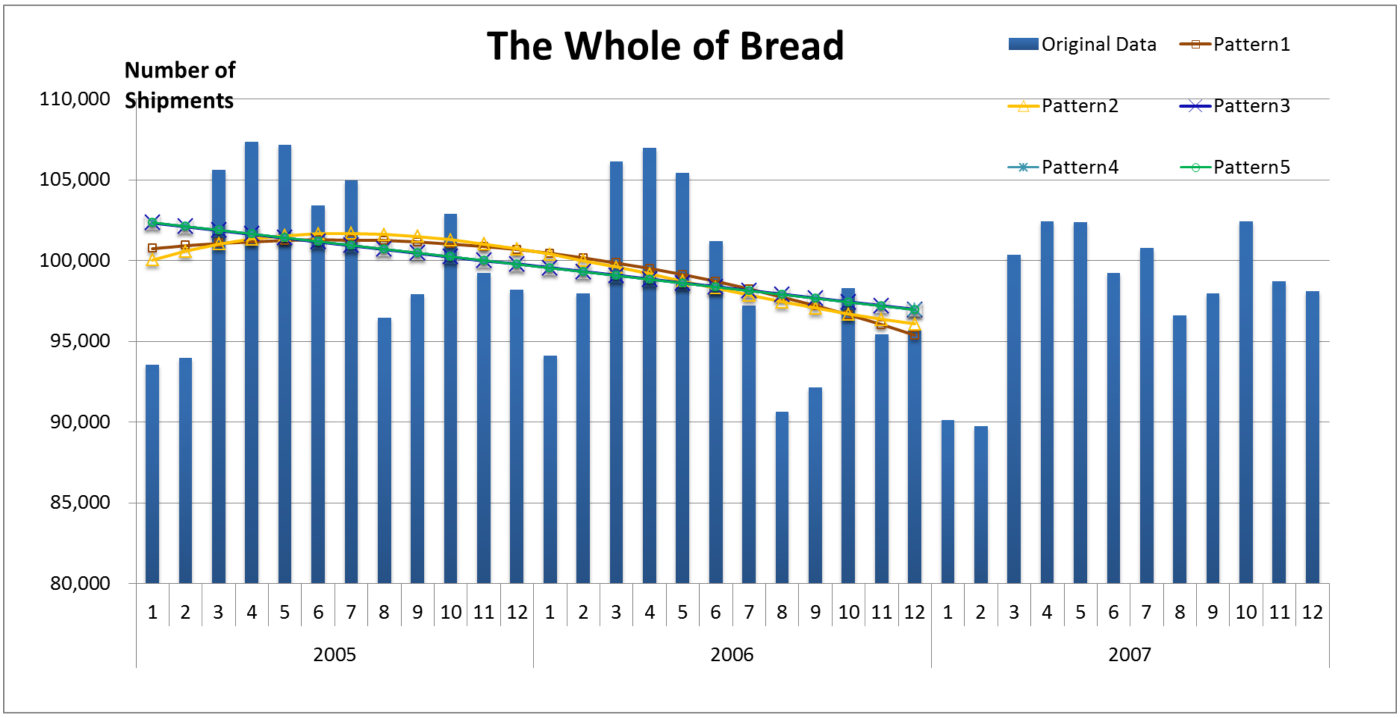

Figure 5-3: Trend of the Whole of Bread

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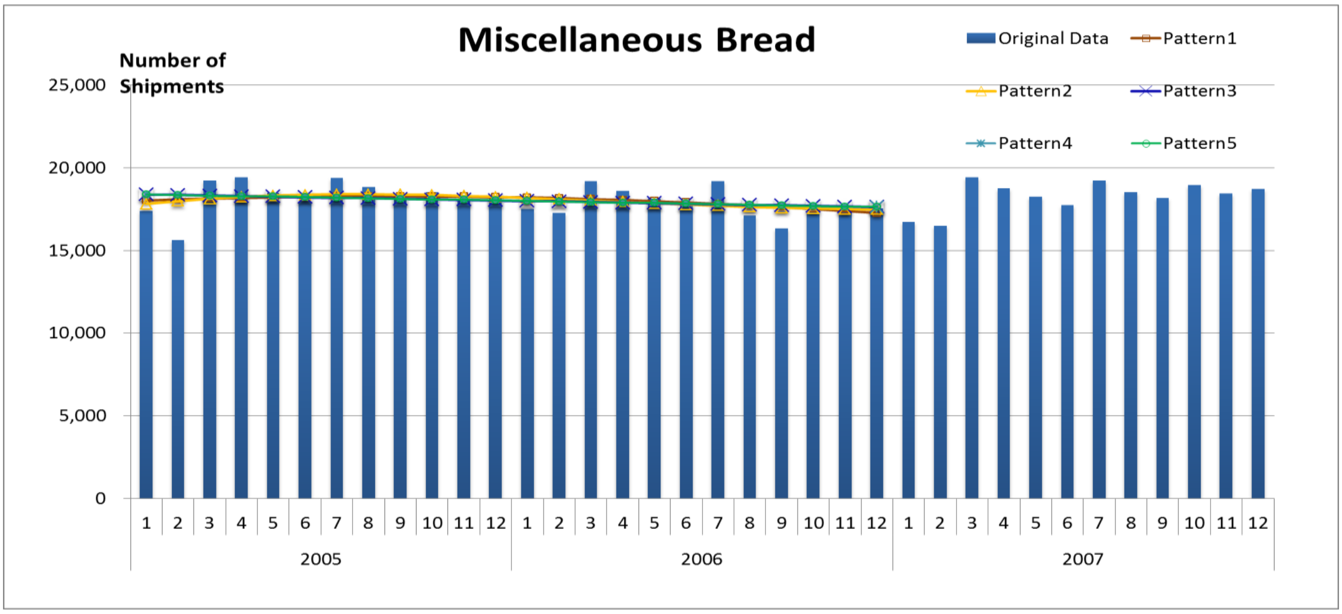

Figure 5-4: Trend of Miscellaneous Bread

5.3 Removing trend of monthly ratio

After removing trend, monthly ratio is calculated by the method stated in 4. Calculation result for 1st to 24th data is exhibited in Table 5-4, 5-5, 5-6, 5-7, 5-8.

Table 5-4: Monthly ratio (Pattern1)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The Whole of Bread	0.93	0.95	1.05	1.07	1.06	1.02	1.01	0.94	0.96	1.02	0.99	0.99
Miscellaneous Bread	0.96	0.91	1.06	1.05	1.01	0.99	1.07	1.00	0.96	1.00	0.99	1.01

Table 5-5: Monthly ratio (Pattern2)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The Whole of Bread	0.94	0.96	1.06	1.07	1.06	1.02	1.01	0.94	0.96	1.02	0.99	0.99
Miscellaneous Bread	0.97	0.91	1.06	1.05	1.01	0.99	1.07	1.00	0.96	1.00	0.99	1.00

Table 5-6: Monthly ratio (Pattern3)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The Whole of Bread	0.93	0.95	1.05	1.07	1.06	1.03	1.02	0.94	0.96	1.02	0.99	0.99
Miscellaneous Bread	0.96	0.91	1.06	1.06	1.01	1.00	1.07	1.00	0.96	1.00	0.99	1.00

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Table 5-7: Monthly ratio (Pattern4)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The Whole of Bread	0.93	0.95	1.05	1.07	1.06	1.03	1.02	0.94	0.96	1.02	0.99	0.99
Miscellaneous Bread	0.96	0.91	1.06	1.06	1.01	1.00	1.07	1.00	0.96	1.00	0.99	1.00

Table 5-8: Monthly ratio (Pattern5)

Month	1	2	3	4	5	6	7	8	9	10	11	12
The Whole of Bread	0.93	0.95	1.05	1.07	1.06	1.03	1.02	0.94	0.96	1.02	0.99	0.99
Miscellaneous Bread	0.96	0.91	1.06	1.06	1.01	1.00	1.07	1.00	0.96	1.00	0.99	1.00

5.4 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (1 6). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (15). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 5-9.

Table 5-9: Estimated Smoothing Constant with Minimum Variance

	Monthly ratio	Pattern1		Pattern2	
		ρ_1	α	ρ_1	α
The Whole of Bread	Used	-0.1471	0.8496	-0.2269	0.7600
	Not used	-0.3675	0.5620	-0.3674	0.5622
Miscellaneous Bread	Used	-0.1682	0.8268	-0.2421	0.7416
	Not used	-0.3317	0.6206	-0.3454	0.5991

	Monthly ratio	Pattern3		Pattern4		Pattern5	
		ρ_1	α	ρ_1	α	ρ_1	α
The Whole of Bread	Used	-0.1073	0.8915	-0.1073	0.8915	-0.1073	0.8915
	Not used	-0.3670	0.5629	-0.3664	0.5639	-0.3670	0.5629
Miscellaneous Bread	Used	-0.1453	0.8515	-0.1453	0.8515	-0.1453	0.8515
	Not used	-0.3190	0.6396	-0.3190	0.6396	-0.3190	0.6396

5.5 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (27). Forecasting results are exhibited in Figure 5-5,5-6 for the cases that monthly ratio is used.

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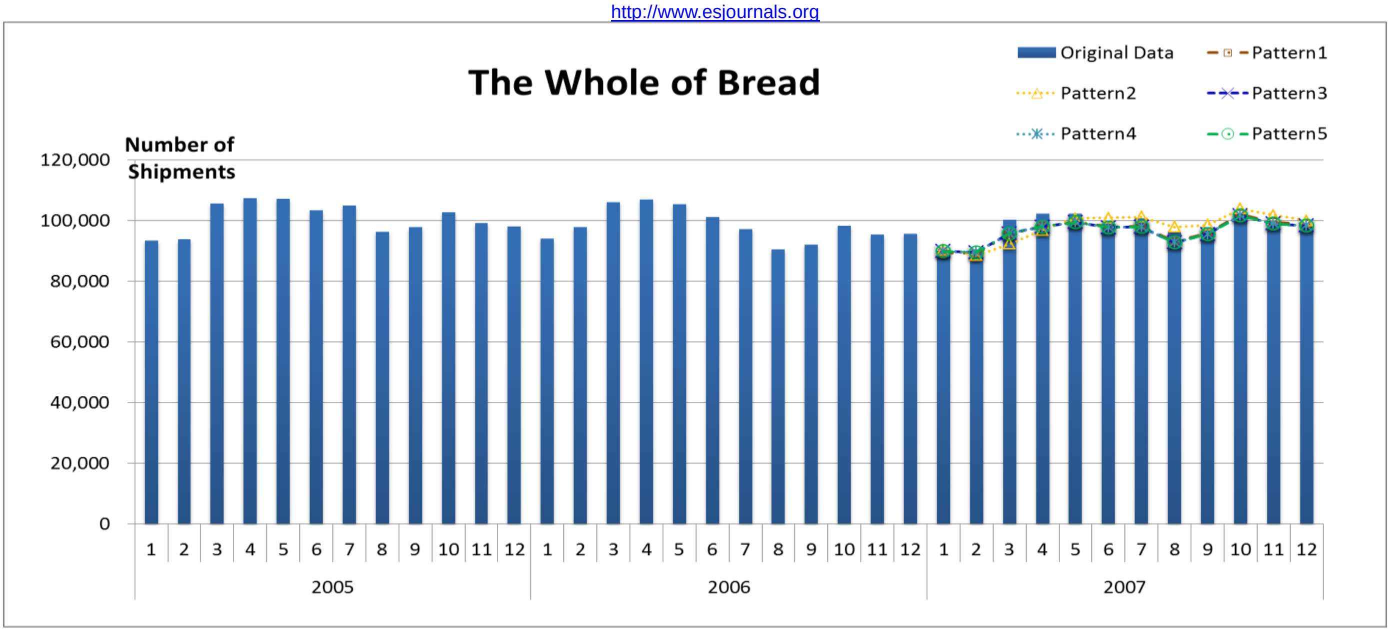

Figure 5-5: Forecasting Results of The Whole of Bread

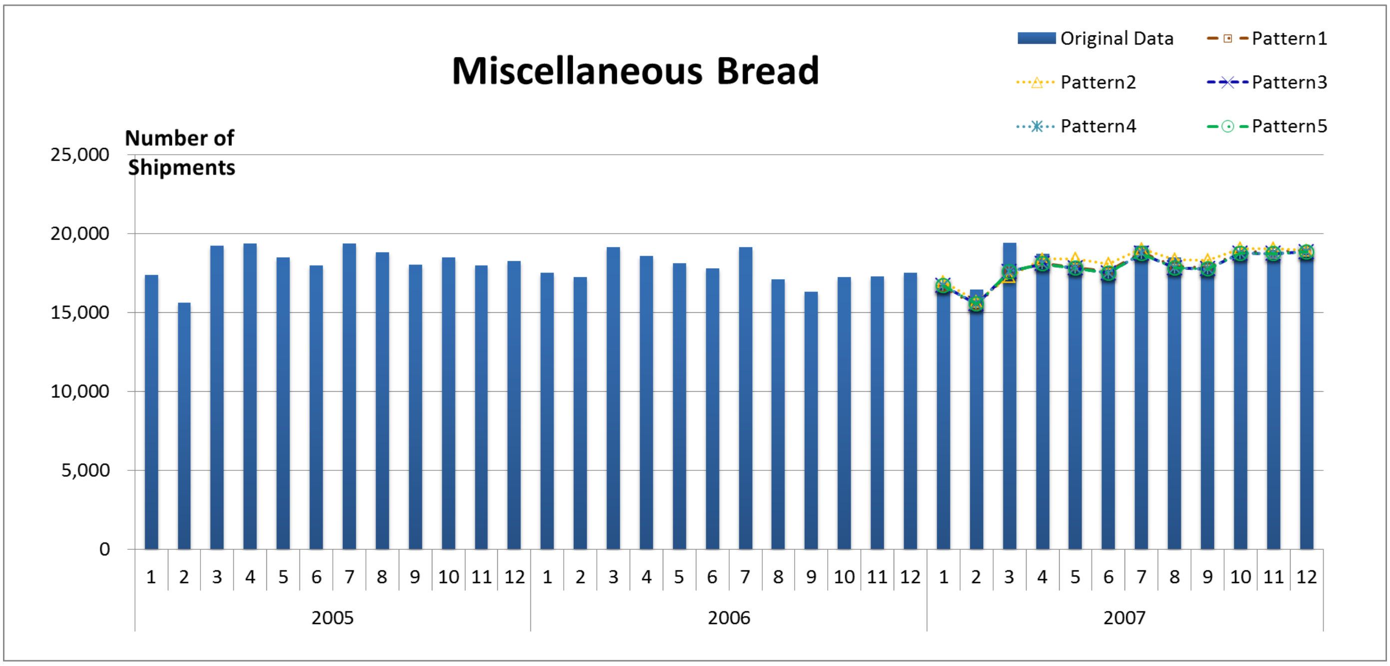

Figure 5-6: Forecasting Results of Miscellaneous Bread

Variance of forecasting error is exhibited in Table 5-10.

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Table 5-10: Variance of Forecasting Error

	Monthly ratio	Pattern1	Pattern2	Pattern3	Pattern4	Pattern5
The Whole of Bread	Used	3,532,009.561	10,686,413.820	3,269,537.146	3,269,537.146	3,269,537.146
	Not used	19,991,995.164	27,673,163.433	19,951,064.607	19,990,834.622	19,951,064.607
Miscellaneous Bread	Used	356,014.221	518,466.768	311,303.913	311,303.913	311,303.913
	Not used	1,114,723.642	1,320,279.708	1,092,240.683	1,092,240.683	1,092,240.683

5.6 Remarks

In all cases, that monthly ratio was used had a better forecasting accuracy. The Whole of Bread had a good result in 1st order, Miscellaneous Bread in 1st order.

6. CONCLUSION

Focusing on the idea that the equation of exponential smoothing method(ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the original production data of bread. The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. In all cases, that monthly ratio was used had a better forecasting accuracy. The Whole of Bread had a good result in 1st order, Miscellaneous Bread in 1st order. Various cases should be examined hereafter.

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