Volume 2 No. 4, April 2012 ISSN 2223-4985

International Journal of Information and Communication Technology Research

http://www.esjournals.org

A Reasonable Test of Expert's Experimental Data

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ABSTRACT

Uncertain statistics is a methodology for collecting and interpreting expert's experimental data by uncertainty theory. And when we collecting the expert's experimental data, the expert's data will maybe affected by psychological factors. In this paper, we are giving a rationality test of expert's experimental data on linear uncertainty distribution of the uncertainty statistics. The testing divided into two parts, one part of testing is to eliminate significant unreasonable data, and the other part is to adjust unreasonable data.

Keywords: *uncertainty theory; uncertain statistics; reasonable test; Wang-Gao-Guo method*

1. INTRODUCTION

There are various types of uncertainty in a real world. This is a motivation to investigate the behavior of uncertain phenomena. Random phenomenon is one of class of objective uncertain phenomena which has been well studied. Besides randomness, fuzziness is another basic type of subjective uncertainty initiated by Zadeh [13] In order to measure a fuzzy event, Liu and Liu[09] introduced credibility measure.

In many cases, fuzziness and randomness simultaneously appear in a system. In order to describe this phenomena, a fuzzy random variable was introduced by Kwakernaak[01,02] , where a random element takes "fuzzy variable" values. This concept was then developed by several researchers such as Puri and Ralescu[12] , Kruse and Meyer[14] , and Liu and Liu[09] according to deferent requirements of measurability. The concept of chance measure of fuzzy random event was first given by Liu [05, 06] Gao and Liu[15] . In addition, a random fuzzy variable was proposed by Liu [16] as a fuzzy element taking "random variable" values. The chance measure for random fuzzy events and expected value for random fuzzy variables were defined by Liu[16] , Liu and Liu[17] , respectively. More generally, a hybrid variable was introduced by Liu[03] as a measurable function from a chance space to the set of real numbers, which is a tool to deal with fuzziness and randomness simultaneously. Essentially, chance theory is a hybrid of probability theory and credibility theory.

Credibility theory founded by Liu [07] and refined by Liu[08] is a branch of mathematics for studying the behavior of fuzzy phenomena. A survey of credibility theory was given by Liu [03] .However, a lot of surveys showed that the subjective uncertainty cannot be modeled by fuzziness. Therefore some real problems cannot be processed by credibility theory. In order to deal with this type of subjective uncertainty, Liu [08] founded an uncertainty theory which is a branch of mathematics based on normality, monotonicity, self-duality, countable subadditivity, and product measure axioms. Since then considerable work has been done based on the uncertainty theory. In order to answer the question how to determine uncertainty distribution of an uncertain variable, Liu present the uncertain statistics is a methodology for collecting and interpreting expert's experimental data by uncertainty theory. The uncertain statistics was then developed by several researchers such as Chen and Ralescu [18] , Wang and Peng [19] , Liu [10] , Hesamian and Taheri[20] , Wang Gao and Guo [21,22] according to different aspects of statistical.

Uncertain statistics is based on expert's experimental data rather than historical data. Evidently, collecting expert's experimental data is one of important work in uncertain statistics. In this aspect, Liu [10] proposed a questionnaire survey for collecting expert's experimental data. The starting point is to invite one or more domain experts who are asked to complete a questionnaire about the meaning of an uncertain variable. So the data which comes from domain expert may contain subjectivity. The subjective also affected by psychological factors. When making a judgment, the expert will be affected by psychological, and then results appeared deviation. Doctor Paul Ekman, an American psychologist, said that if man was lying, it is difficult to put a lie out flashbacks. Inspired by the principle, we create a reasonable test in this paper. Of course, the expert who will be asked does not lie, the test just to reduce interference by the psychological.

In this paper, the reasonable test will be introduced. We organize this paper as follows. In the next section, we mention some concepts in uncertainty theory and Psychology theorems required in the paper. In Section 3, some basic concepts of uncertain statistics are recalled such as Wang-Gao-Guo[22] Method. In Section 4, we introduce the reasonable test from two parts, one is refusing significantly unreasonable expert's experimental data, and another is adjusting significantly unreasonable expert's experimental data. Finally, a conclusion is drawn in Section 5.

2. PRELIMINARIES

In this section, we will introduce some useful definitions about uncertain variables and linear uncertainty distribution, and also we will mention several psychology common senses.

Let Γ be a nonempty set, and let $\mathcal{L}$ be a algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is called an event. In order

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to measure uncertain event, uncertain measure $\mathcal{M}$ was introduced as a set function satisfying the following four axioms:

Axiom 1. *(Normality)* $\mathcal{M}\{\Gamma\}=1$.

Axiom 2. *(Increasing)* $\mathcal{M}\{\Lambda_1\}\le\mathcal{M}\{\Lambda_2\}$ *whenever* $\Lambda_1\subset\Lambda_2$.

Axiom 3. *(Self-Duality)* $\mathcal{M}\{\Lambda\}+\mathcal{M}\{\Lambda^c\}=1$ *for any event* Λ.

Axiom 4. *(Countable Subadditivity) For every countable sequence of events* $\{\Lambda_i\}$ *, we have*

$$\mathcal{M}\{\bigcup_{i=1}^{\infty}\Lambda_i\}\le\sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Then the triplet $(\Gamma,\mathcal{L},\mathcal{M})$ is called an uncertainty space. An uncertain variable is defined as a measurable function ξ from an uncertainty space $(\Gamma,\mathcal{L},\mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real set $\Re$, the set $\{\xi\in B\}=\{\gamma\in\Gamma\mid\xi(\gamma)\in B\}$ is an event.

Extra definitions may be introduced when needed. Such as follows:

Definition 1: (Liu[22]) The uncertainty distribution $\Phi:\Re\to[0,1]$ of an uncertain variable ξ is defined by $\Phi(x)=\mathcal{M}\{\xi\le x\}$.

Distribution 2: (Liu[23]) An uncertain variable ξ is called linear if it has a linear uncertainty distribution

$$\Phi(x)=\begin{cases}0 & if \quad x\le a\\ (x-a)/(b-a) & if \quad a\le x\le b\\ 1 & if \quad x\ge b\end{cases}$$

Denoted by $\mathcal{L}(a,b)$ where a and by are real numbers with $a\le b$.

3. UNCERTAIN STATISTICS

Uncertain statistics is a methodology for collecting and interpreting expert's experimental data by uncertain theory. This section will introduce a questionnaire survey for collecting expert's experimental data designed by Liu\cite{10} , and introduce Wang-Gao-Guo\cite{22} method, one of uncertain hypothesis testing.

Collecting expert's experimental data: For example. We give a questionnaire about the meaning of an uncertain variable ξ like "how far from Beijing to Tianjin". We first ask the domain expert to choose a possible value x that the uncertain variable ξ may take. Then, we quiz him "how likely is ξ less than x" Denote his belief degree by α . Thus we obtain an expert's experimental data (x,α) from the domain expert. Repeating the above process, we obtain the expert's experimental data.

Let $(x_1,\alpha_1),(x_2,\alpha_2),...,(x_n,\alpha_n)$ be the expert's experimental data that meet the following condition $x_1<x_2<\cdots<x_n, 0\le\alpha_1\le\alpha_2\le\cdots\le\alpha_n$

Principle of least squares: Assume that an uncertainty distribution to be determine has a known functional form with one or more unknown parameters like $\Phi(x;\theta_1,\theta_2,...,\theta_p)$ where $\theta_1,\theta_2,...,\theta_p$ are unknown parameters. How to estimate those unknown parameters? Liu presented the principle of least squares which says that the unknown parameters $\theta_i, i=1,2,...,p$ are the solution of the minimization problem,

$$\min_{\theta_1,...,\theta_p}\sum^{n}\left(\Phi(x_i;\theta_1,\theta_2,...,\theta_p)-\alpha_i\right)^2.$$

If the uncertainty distribution $\Phi(x;\theta_1,\theta_2,...,\theta_p)$ is regular, then Φ^{-1} is the inverse uncertainty distribution of ξ. It means that the principle of least squares can be changed into the following form.

$$\min_{\theta_1,...,\theta_p}\sum^{n}\left(\Phi^{-1}(\alpha_i;\theta_1,\theta_2,...,\theta_p)-x_i\right)^2.$$

where the unknown parameters $\theta_i, i=1,2,...,p$ are the solution of the minimization form.

Linear uncertainty distribution: (Liu[25]) Assume that an uncertainty distribution has a linear form with two unknown parameters a and b , i.e.,

$$\Phi(x)=aX+b$$

Based on expert's experimental data $(x_1,\alpha_1),(x_2,\alpha_2),...,(x_n,\alpha_n)$, the unknown parameters a ,b should solve

$$\min_{a,b}\sum_{i=1}^{n}(ax_i+b-\alpha_i)^2$$

The linear uncertainty distribution is $\Phi(x)=\hat{a}X+\hat{b}$ where

Volume 2 No. 4, April 2012 ISSN 2223-4985

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$$\hat{a}=\frac{nx^{*}\alpha^{*}-\sum_{i=1}^{n}x_i\alpha_i}{nx^{*2}-\sum_{i=1}^{n}x_i^2},\qquad \hat{b}=\alpha^{*}-\hat{\alpha}x^{*}$$

$$x^{*}=\left(x_1+x_2+\cdots+x_n\right)/n,\qquad \alpha^{*}=\left(\alpha_1+\alpha_2+\cdots+\alpha_n\right)/n$$

Wang-Gao-Guo Method :(Liu[24]) This method of uncertain hypothesis testing was proposed by Wang, Gao and Guo[21] for determining if the views of two domain experts are identical (i.e., whether or not they have the same uncertainty distribution).

Denote the two domain experts by A and B. From them, we acquire their expert's experimental data from which two empirical uncertainty distribution Φ and Ψ are product. Now let us randomly generate two sequences of pairs,

$$A:\left(x_1,\alpha_1\right),\left(x_2,\alpha_2\right),...,\left(x_n,\alpha_n\right)$$
$$B:\left(y_1,\beta_1\right),\left(y_2,\beta_2\right),...,\left(y_n,\beta_n\right)$$

Such that

$$\Phi\left(x_i\right)=\alpha_i,\qquad \Psi\left(y_i\right)=\beta_i$$

For $i=1,2,...,m$ and $j=1,2,...,n$.

Then we combine $x_1,x_2,...,x_m$ with $y_1,y_2,...,y_n$ to form a first new sequence of size $m+n$. After a rearrangement from small to large, we get $AABAB\cdots B$. Where A indicates that the data at this position if from $x's$, and B indicates that the data at this position if come $y's$. By the same way, we will get second new sequence $ABBAA\cdots B$ that come from the combination $\alpha_1,\alpha_2,...,\alpha_m$ with $\beta_1,\beta_2,...,\beta_n$.

Next let useful compare the two sequences (i.e., first new sequence and second new sequence) and obtain a 0-1 square:

$01001\cdots0$

Where 0 indicates that the two letters at this position are identical and 1 indicates that the two letters at this position are different.

In order to determine if the two letters at this position are identical, we have the following two hypotheses:

H_0 : *Two domain experts have the same view*;

H_1 : *Two domain experts have different views*.

We will call H_0 a null hypothesis and H_1 an alternative hypothesis.

If the null hypothesis H_0 is true (i.e., the two domain experts have the same uncertainty distribution), then the number T of 1's in the 0-1 sequences should not be too large. Thus the decision rule is to reject the null hypothesis H_0 if

$$T\geq\rho\left(m+n\right)$$

Where ρ is a predetermined confidence level, for example, $\rho=0.2$.

4. THE REASONABLE TEST

4.1 Refusing Significant Unreasonable Expert's Experimental Data

Wang-Gao-Guo method is a wonderful hypothesis testing for judging whether or not the views of two domain experts are the same uncertainty distribution. But we have a new question is how to discuss the character of single domain expert's experimental data, such as the reasonable of experimental data. Reasonable test will answer this question.

Assume that uncertain variable ξ is linear, and denote the single domain expert's experimental data by A,

$$A:\left(x_1,\alpha_1\right),\left(x_2,\alpha_2\right),...,\left(x_n,\alpha_n\right)$$

And then, we form a new sequences depend on A. Denote the new sequences by A',

$$A':\beta_1,\beta_2,...,\beta_{n-1}$$

Where

$$\alpha_1<\beta_1<\alpha_2,...,\alpha_i<\beta_i<\alpha_{i+1},...,\alpha_{n-1}<\beta_{n-1}<\alpha_n$$

So the size of A' is n-1.

Then quiz the same expert in line with sequences A' by:

"If I give β_i to be your belief degree, the ξ less than or equal to which number is appearing in your heart by the moment."

Denote the expert's answer by y_i. Repeating the above process, the following expert's experimental data are obtained by questionnaire. And a new expert's experimental data appeared. Denote this new expert's experimental data by B.

$$B:\left(y_1,\beta_1\right),\left(y_2,\beta_2\right),...,\left(y_{n-1},\beta_{n-1}\right)$$

We will call B auxiliary data.

According to the Wang-Gao-Guo method, we combine $x_1,x_2,...,x_n$ with $y_1,y_2,...,y_{n-1}$ to form a new sequence of size $2n-1$. After a rearrangement from small to large, we get

$$ABABB...A \tag{1}$$

Where A indicates that the data at this position is from $x's$, and B indicates that the data at this position is from

Volume 2 No. 4, April 2012 ISSN 2223-4985
International Journal of Information and Communication Technology Research

http://www.esjournals.org

$y's$. We also combine $\alpha_1, \alpha_2, ..., \alpha_n$ with $\beta_1, \beta_2, ..., \beta_{n-1}$ to form another new sequence of 2n-1. After a rearrangement from small to large, we get

$$ABABA...A \quad (2)$$

Where A indicates that the data at this position is from $\alpha's$, and B indicates that the data at this position is from $\beta's$.

Next let us compare the two sequences (1) and (2) and obtain a 0-1 sequence:

$$00001...0 \quad (3)$$

Where 0 indicates that the two letters at this position are identical and 1 indicates that the two letters at this position are different.

In order to determine if the expert's experimental data is reasonable, we have the following two hypotheses:

H_0 : *The expert's experimental data have no significant unreasonable;*

H_1 : *The expert's experimental data havesignificant unreasonable;*

If the null hypothesis H_0 is true, then the number T of $1's$ in the sequence (3) should not be too large. Thus the decision rule is to reject the null hypothesis H_0 if

$$T \ge \rho(2n-1)$$

Where is a predetermined confidence level [21],

$$\rho = \frac{number\ of\ 1\ in\ the\ 0-1\ sequences}{2n-1}$$

for example, $\rho = 0.2$.

If we reject the null hypothesis H_0, which means that the expert's experimental data exists significant unreasonable, and we have enough grounds for refusing to use this expert's experimental data, else we keep going to d next part of reasonable test.

4.2 Adjusting Unreasonable Expert's Experimental Data

When we accept the null hypothesis H_0, that meaning the expert's experimental data have no significant unreasonable, and also the expert's experimental data may exist unreasonable. The purpose of this part of reasonable test is to find unreasonable data in the expert's experimental data A, and then adjust it.

We combine A with B to form a matrix R and do merge sort here from left to the right according to the α_i and β_i,

$$R = \begin{pmatrix} x_1 & y_1 & x_2 & ... & y_{n-1} & x_n \\ \alpha_1 & \beta_1 & \alpha_2 & ... & \beta_{n-1} & \alpha_n \end{pmatrix}$$

According to the R, we form vector quantities

$$\vec{n}_i = (y_i - x_i, \beta_i - \alpha_i)$$

Where

$i = 1, 2, ..., n-1$. and

$\vec{n}_n = (x_n - y_{n-1}, \alpha_n - \beta_{n-1})$. Then the included angle u_i of adjacent vector quantity meets the following,

$$k_i = \cos(u_i) = \frac{\vec{n}_i \cdot \vec{n}_{i+1}}{|\vec{n}_i||\vec{n}_{i+1}|} \quad (4)$$

So we write a data sequence as

$$k_1, k_2, k_3, .., k_{2n-3}$$

The next step is to find this group of unusual values in this sequence. As we known, if linear uncertainty distribution is $\Phi(x) = aX + b$, k_i is random variable, and its distribution is normal distribution, note that

$$k_i \sim N(1, \sigma^2)$$

Where σ is unknown parameter, so we obtain statistic

$$g = \frac{k_i - \mu}{s/\sqrt{2n-3}}$$

and the statistic obeyed the law of t-distribution

$$g \sim t(2n-4)$$

Where $s^2 = \frac{1}{2n-4}\sum_{i=1}^{2n-3}(k - \bar{k})^2$

n order to determine if the expert's experimental data is reasonable, we have the following two hypotheses:

H_0 : *The expert's experimental data is reasonable;*

H_1 : *The expert's experimental data exit unreasonable.*

If the expert's experimental data is reasonable, each k_i is steadily fluctuations. And $|k_i - 1|$ will not be too large. Hence the decision rule is to reject the null hypothesis H_0 if

$$k_i > s \cdot t_{1-\eta/2} \cdot s + \bar{k} \text{ or } k_i < \bar{k} - t_{1-\eta/2} \cdot s$$

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Where $P\left(|k-1|>t_{1-\eta/2}\right)=\eta/2$, η is a predetermined confidence level, for example, η=0.05.

Now, we will adjust the expert's experimental data if we reject the null hypothesis H_0, Assume the k_i is a unusual values, then we will find corresponding vector quantities according to (4) as follow,

$\vec{n}_i=\left(y_i-x_i,\beta_i-\alpha_i\right)$ $\vec{n}_{i+1}=\left(x_{i+1}-y_i,\alpha_{i+1}-\beta_i\right)$

Further, we will find three groups of data, influencing value of k_i, from A an B as follow,

$$\left(x_i,\alpha_i\right),\left(y_i,\beta_i\right),\left(x_{i+1},\alpha_{i+1}\right)$$

Hence unreasonable data $\left(x_i,\alpha_i\right),\left(x_{i+1},\alpha_{i+1}\right)$ from expert's experimental data have been found, and then we adjust the unreasonable data by

$$\left(x_i',\alpha_i'\right)=\left(\frac{x_i+y_i}{2},\frac{\alpha_i+\beta_i}{2}\right),$$

$$\left(x_{i+1}',\alpha_{i+1}'\right)=\left(\frac{x_{i+1}+y_{i+1}}{2},\frac{\alpha_{i+1}+\beta_{i+1}}{2}\right)$$

So a new expert's experimental data is appearing. Denote it by $\hat{A}$

$$\hat{A}:\left(x_1,\alpha_1\right),\ldots,\left(x_i',\alpha_i'\right),\left(x_{i+1}',\alpha_{i+1}'\right),\ldots,\left(x_n,\alpha_n\right)$$

We will use $\hat{A}$ to determine uncertainty distribution by principle of least squares or other means.

Example: Forecasting the Average Scores of Chemical Examination

In this example, we invite a chemical teacher to analyze the degree of difficulty of a Chemical exam in December 2011. The teacher estimates the average scores and his belief degree on the basic of his knowledge and experience, and we will use these experimental data to forecast the real average of this examination, before that, we should test whether this experimental data are reasonable or not.

The teacher's experimental data is as below,

$$A(x_i,\alpha_i):(58,0)(56,0.2)(58,0.5)(60,0.6)(63,0.9)(65,1)$$

And also, we got the test data is as below,

$$B(y_j,\beta_j):(56,0.1)(57,0.4)(59,0.55)(63,0.7)(64,0.95)$$

We present the hypothesis as follows:

H_0: *The expert's experimental data have no significant unreasonable;*

H_1: *The expert's experimental data have significant unreasonable;*

Then we combine x_i with y_j to form a first new sequence of size $2n-1$. After a rearrangement from small to large, we get,

$$A\ B\ A\ B\ A\ B\ A\ A\ B\ B\ A$$

We also combine α_i with β_j to form another new sequence. After a rearrangement from small to large, we get,

$$A\ B\ A\ B\ A\ B\ A\ B\ A\ B\ A$$

So we obtain the 0-1 sequence as follows:

$$00000001100$$

Let T denote the number of 1 in this sequences. Since $T=2<2.2=\rho(2n-1)$, we accept the *null hypothesis* H_0 by our decision rule. Therefore, we believe that the teacher's experimental data have no significant unreasonable under the level $\rho=0.2$, and $T\neq 0$, which means that the teacher's experimental data also exist unreasonable, so we need to adjust unreasonable expert's experimental data.

"The average scores of Chemical exam" is an uncertain variable, and assuming that the uncertainty distribution of this uncertain variable is a linear, then we combine A with B to form a matrix R and do merge sort here from left to the right according to the α_i and β_i,

$$R=\begin{pmatrix}55 & 56 & 56 & 57 & 58 & 59 & 60 & 63 & 63 & 64 & 65\\ 0 & 0.1 & 0.2 & 0.4 & 0.5 & 0.55 & 0.6 & 0.7 & 0.9 & 0.95 & 1\end{pmatrix}$$

And we got vector quantities as follows,

$$\vec{n}_1=(1,0.1)\quad \vec{n}_2=(0,0.1)\quad \vec{n}_3=(1,0.2)\quad \vec{n}_4=(1,0.1)\quad \vec{n}=(1,0.05)$$
$$\vec{n}_6=(1,0.05)\quad \vec{n}=(3,0.1)\quad \vec{n}_8=(0,0.2)\quad \vec{n}=(1,0.05)\quad \vec{n}_{10}=(1,0.05)$$

So the angle of each vector can be found, and the cosine value of them also be obtained as below,

$$k=(0.0995\quad 0.1961\quad 0.9952\quad 0.9988\quad 1.0000\quad 0.9999\quad 0.0333\quad 0.0499\quad 1.0000)$$

As previously mentioned, k_i is random variable, and its distribution is normal distribution, note that

$$k_i\sim N(1,\sigma^2)$$

Where σ is unknown parameter. we get the statistic as below:

$$\vartheta=\frac{k_i-1}{s/3}$$

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Where $s^2 = \frac{1}{8}\sum_{i=1}^{9}(k_i - 1)^2 = 0.4118.$ we have the following two hypotheses:

H_0 : *The expert's experimental data is reasonable*;

H_1 : *The expert's experimental data exit unreasonable*;

Hence the decision rule is to reject the null hypothesis H_0 if

$$k_i > 1.5232 \quad or \quad k_i < 0.4768$$

When $\eta = 0.05$ .Check the data k , and get the result is as follows:

$$\{k_i \mid k < 0.4768\} = \{k_1, k_2, k_7, k_8\}$$

So we obtain the unreasonable data as follow:

$$A:(55,0)(56,0.2)(60,0.6)(63,0.9) \quad B:(56,0.1)(57,0.4)(63,0.9)(64,0.95)$$

Then we adjust the unreasonable data, we get $\hat{A}$:

$$\hat{A} := (55.5,0.05)(56.5,0.3)(58,0.5)(61.5,0.75)(63.5,0.925)(65,1)$$

Assume that uncertainty distribution has a linear form with two unknown parameters a and b . Separately in A and B do the least square method operations to estimate a and b , we get the estimate value as below:

$$A: a = 0.0968, b = -5.2247; \hat{A}: a = 0.0933, b = -5.0079$$

In order to avoid awkward phrasing, we use the pronoun ξ in describing the uncertainty value "The average scores of Chemical exam". In the uncertainty theory, expected value is the average of uncertain variable in the sense of uncertain measure, and represents the size of uncertain variable. ξ is an uncertain variable, then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge r\}dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \le r\}dr$$

Hence we get each expected value based on A and $\hat{A}$

$$A: E(\xi) = 59.156; \hat{A}: E(\xi) = 59.06$$

After this investigation, the average of the Chemical exam is 58.6.

5. CONCLUSIONS

The reasonable test of expert's experimental data is to detect if the data that come from expert is profession and steady or not. The expert's experimental data A is use to estimate the uncertainty distribution, and the expert's data B is use to test the expert's data A . This method depend on the principle of the reasonableness of the expert's experimental data, some example shows that this method works well.

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