

Concatenated Convolutional Codes for Deep Space Mission

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ABSTRACT

In satellite communication deep space mission are the most challenging mission, where system has to work at very low E_b/N_0 . Concatenated codes are the ideal choice for such deep space mission. ISRO is planning to send unmanned mission for Mars and several deep space missions in future to study and detailed understanding of our own universe. This paper basically reviews the concatenated convolutional code structure and selects the suitable candidate for our future deep space mission. The complete simulation using Simulink is done and results are presented in this paper.

Keywords: E_b/N_0 , PCCC, HCCC, SCCC, SNR, AWGN,

1. INTRODUCTION

The usefulness of concatenated codes was first noticed by Forney in [1]. In general, the concatenation of convolutional codes can be classified into three categories, i.e., PCCC, SCCC and hybrid concatenated convolutional codes (HCCC). The constituent convolutional codes (CCs) used in each scheme fall into several classes of systematic, nonsystematic, recursive and non-recursive schemes. Systematic convolutional codes have their inputs appear directly at the output, while non systematic convolutional codes do not have this property. A non-recursive encoder does not have any feedback connection while a recursive encoder does. In general, nonsystematic non-recursive CCs perform almost the same as equivalent systematic recursive CCs since they exhibit the same distance spectrum. In the original turbo code, two identical recursive systematic convolutional (RSC) codes were used. Several other authors have explored the use of nonsystematic recursive CCs as the constituent codes, e.g., Massey and Costello [2, 3]. In [4, 5], Benedetto et al. and Perez et al. showed that recursive CCs can produce higher weight output codewords compared to nonrecursive CCs, even when the input information weight is low. This is a major advantage in a PCCC system since low input weight codewords dominate the error events. In addition, PCCC requires a long information block in order to perform well in the low SNR region. In this case, recursive CCs can provide an additional interleaving gain that is proportional to the length of the interleaver while nonrecursive CCs cannot. Therefore, RSCs are preferable in practice as the constituent code for a PCCC or the inner code for an SCCC or HCCC. Detailed treatments of the constituent CC encoder can be found in Lin and Costello [6] and many excellent references within, e.g., [4, 7]. In the following sections, we will examine the structure for each scheme. We assume that these systems consist of only two CCs. Extension to multiple CCs is straightforward and have been investigated in a number of references [8, 9].

2. PARALLEL CONCATENATED CODES

Parallel-Concatenated Convolutional Codes (PCCC), known as *turbo codes*, allow structure through concatenation and randomness through interleaving. The introduction of turbo codes has increased the interest in the coding area since these codes give most of the gain promised by the channel-coding theorem.

The CCSDS Telemetry Channel Coding Recommendation [1] establishes a common framework and provides a standardized basis for the coding schemes used by CCSDS Agencies for space telemetry data communications. This standard traditionally provides the benchmark for new and emerging coding technologies. Turbo codes have an astonishing performance of bit error rate (BER) at relatively low E_b/N_0 . Turbo codes were chosen as a new option for this standard in 1999, only 6 years since their official presentation to the international community: this was the first international standard including turbo codes. The reason was the significant improvement in terms of power efficiency assured by turbo codes over the old codes of the standard. Figure.1 shows complete SIMULINK model of CCSDS compliant turbo encoder and decoder.

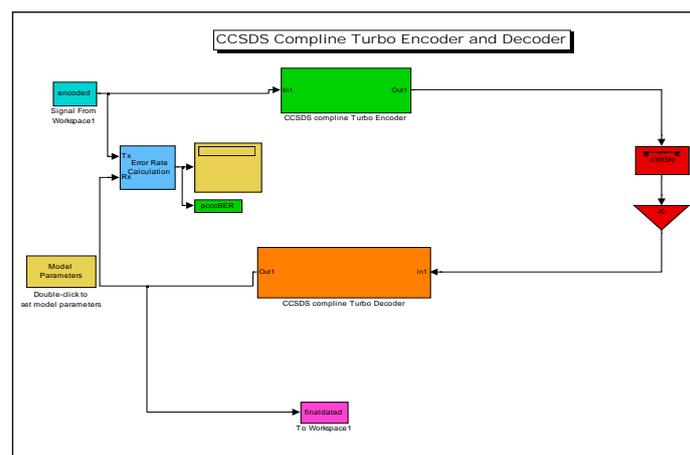


Figure.1 CCSDS Compliant Turbo Encoder and Decoder

2.1 Turbo Encoder and Decoder

In this case, two RSCs of rates $R_i = 1/n_i$ and $i \in \{1, 2\}$ are connected in parallel. The interleaver π interleaves the uncoded message $\mathbf{u} = \{u_0, u_1, \dots, u_{N-1}\}$, of length N before entering into the second encoder. If the constituent encoders are RSC codes and no termination of the constituent codes is performed, the overall code rate R for this PCCC scheme is:

$$R = \left(\sum_{i=1}^2 \frac{1}{R_i} - 1 \right)^{-1} \quad (1)$$

Clearly, the code rate R in (1) is less than the individual code rate R_n of each constituent encoder. For example, the PCCC in [10] uses two identical constituent RSC encoders of rate 1/2 each. Since the two RSC encoders produce the same original message \mathbf{u} at the output (one interleaved and one not interleaved), one of them is therefore deleted. Applying (1), the overall code rate is equal to 1/3.

This low rate system offers very strong protection to the transmitted message. Generally, the lower the code rate, the higher the protection to the transmitted data. In practice, a low code rate system is used when the SNR is low or the bandwidth is large. However, a low code rate is inefficient in a bandwidth limited system due to the extra redundancy in the coded message. A higher coding rate is necessary for achieving higher bandwidth efficiency.

Optimally, a high code rate concatenated system should use high rate constituent CCs with the largest effective distance d_{eff} , where d_{eff} is the smallest Hamming weight of codewords with input weight two [11]. However, due to the constraints of the decoder, i.e., the trellis branch complexity increasing almost exponentially with respect to the input into the encoder, implementation of these systems are not normally used in practice. In past, several authors have tried to use the dual code [12, 13] to design very high rate turbo codes with low decoder complexity. However, the implementation of the decoder is not easy because the estimation of the branch and state metrics in the decoder requires a very high level of accuracy.

A simple technique to obtain a higher code rate using the same low rate constituent code is called puncturing. Referring to Figure 2, certain parity bits (C11 and C22) are deleted from the encoded sequence before going into the multiplexer. The advantage of this puncturing technique is that it requires no changes in the decoder, i.e., the same rate 1/2 decoders can be used for different higher code rates. This is especially useful in an adaptive system where code rates need to be varied depending on the channel conditions. The penalty to pay for puncturing a low rate encoder to a higher rate encoder is that the system performance is degraded in comparison to a similar high rate encoder without puncturing. This is due to a lower d_{eff} or a larger number of effective nearest neighbours N_{eff} of the punctured code. In addition, when RSC codes are used, there are two choice either deletion of the parity bits or deletion or deletion of systematic bits. However in general deletion of parity bit is compared to

systematic bits is preferred. This restricts us from choosing an optimal puncturing matrix for a very high code rate, e.g., $k/(k + 1)$, since many parity bits are required. In this paper we will compare the performance of both type of puncturing structure.

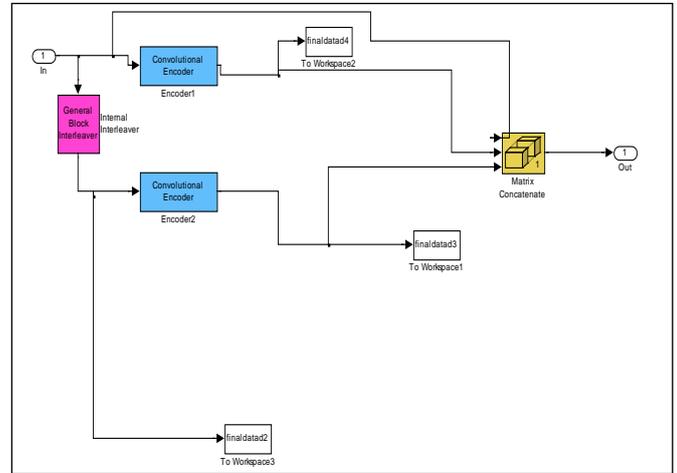


Figure.2 CCSDS compliant Turbo Encoder

To investigate the “goodness” of turbo code performances, it is useful to compare them against the channel coding theoretical limits. For a fixed code-rate k/n and a specific constellation, the ideal spectral efficiency η (measured in bps/Hz) is computed by referring to ideal Nyquist base-band filters. For 4-PSK constellations the following expression results:

$$\eta = 2 \frac{k}{n}$$

Consider the transmission of a binary turbo code over the AWGN channel by a Gray labelled 4-PSK. At very high signal-to-noise ratios (SNR), that is very low error rates, the code performance practically coincides with the union bound, truncated to the contribution of the minimum distance. The FER and BER code performance can then be approximated by:

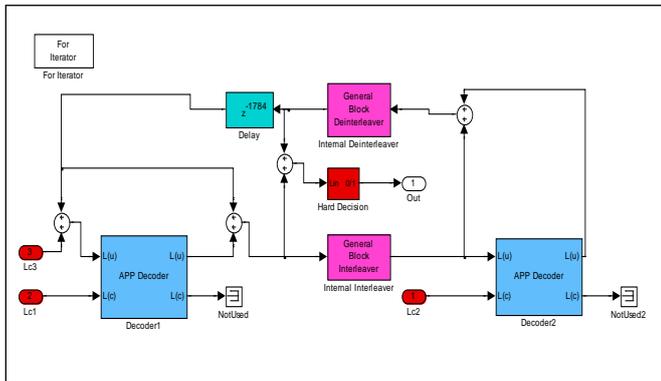
$$FER \cong \frac{1}{2} A_{\text{min}} \operatorname{erfc} \left(\sqrt{d_{\text{min}} \frac{k}{n} \frac{E_b}{N_o}} \right) \quad (2)$$

$$BER \cong \frac{1}{2} \frac{w_{\text{min}}}{k} \operatorname{erfc} \left(\sqrt{d_{\text{min}} \frac{k}{n} \frac{E_b}{N_o}} \right) \quad (3)$$

Where A_{min} is the code *dominant multiplicity* (number of codewords with weight d_{min}), and w_{min} is the code *dominant information multiplicity* (sum of the Hamming weights of the A_{min} information frames generating the codewords with weight d_{min}). When comparing the simulated curves with Eq. 2 and 3 a small fixed penalty (usually less than 0.25 dB for

turbo codes) must be also taken into account, due to the sub-optimality of iterative decoding.

Figure 3. shows a SIMULINK model of CCSDS comply Turbo decoder, Here puncturing is done on parity bits . The puncturing matrix in this case is [1 0 1 1 1 0] . Figure 4 shows a SIMULINK model of a turbo decoder, where systematic bits are not send from encoder side. So there is no puncturing and depuncturing involve. Figure 5 shows the comparative performance of two cases. Result shows that deletion of parity bit will be preferred over



systematic bits.

Figure 3 CCSDS Comply Turbo Decoder

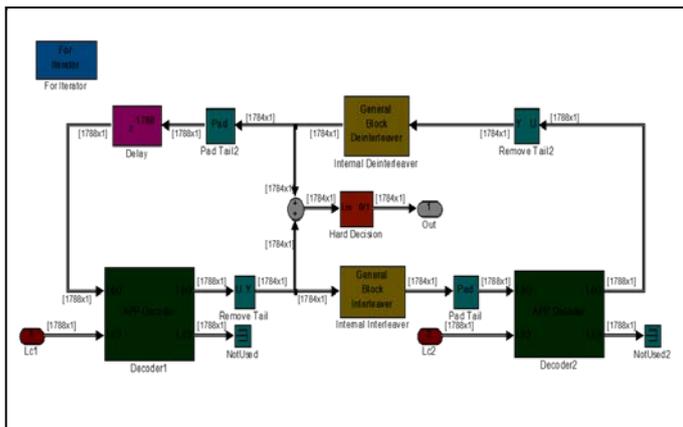


Figure 4: Turbo Decoder without Transmission of Systematic Bits

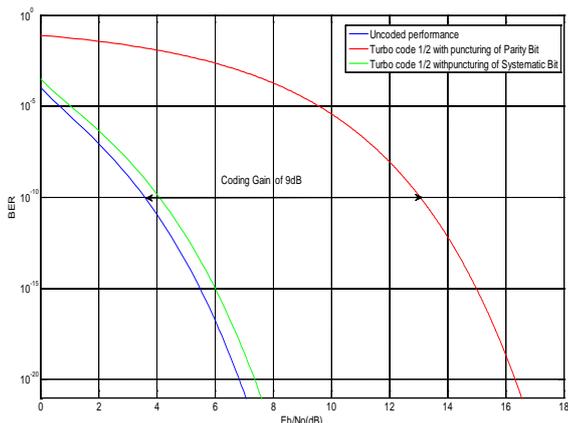


Figure 5: Comparative Performance Analysis of Different Turbo Codecs

3. SCCC ENCODER AND DECODER

The breakthrough idea of parallel turbo codes in [14] has attracted a vast body of research into serial concatenated schemes. S. Benedetto, who in 1996 proposed serial concatenated convolutional codes (SCCC) concept [4]. SCCC combine Forney’s serial concatenated codes’ (RS codes + convolutional code) and PCCCs’ (Turbo code) features, in an appropriate signal to noise ratio range, through iterative decoding can achieve very excellent decoding performance. In September 2007 CCSDS131.2-O-1 presented using the flexible serial concatenated convolutional Turbo codes, namely SCCC code in telemetry systems which made performance close to the Shannon limit. SCCCs were found to outperform PCCC at high SNR in terms of both bit error probability and frame error probability.

The main difference between Forney’s concatenated scheme [1] and SCCC is that a random interleaver is used in SCCC. This has certain advantages in terms of randomizing the burst errors and allows an iterative exchange of soft information between the inner and outer decoders in the receiver. Compared to PCCC schemes, the encoding process in SCCC is different. The original message u is first encoded by the outer encoder E_0 . Unlike PCCC where the input into the next encoder is the interleaved version of the same message u , the inner encoder E_1 in SCCC uses the interleaved coded sequence from E_0 as its input. It is noted that in SCCC, only the inner encoder is required to be RSC to achieve interleaver gain [10]. Similar to the PCCC scheme, a higher code rate in the SCCC can be obtained from puncturing a low rate constituent encoder or using high rate CCs. The overall code rate for the system in Figure 6 without code termination is:

$$R = R_1 R_0$$

where R_1 and R_0 are the code rate of E_1 and E_0 , respectively. In practice, some SCCC schemes use an outer block code to the SCCC structure [15–17]. The function of the outer code is to correct the burst errors and improve the performance in the error floor region. Common classes of block codes used are Bose, Chaudhuri and Hocquenghem (BCH) and Reed Solomon (RS) [18, 19]. Performance of this system is not covered in this paper.

It was shown in [20, 21] that there is a great difference in the interleaver gain between PCCC and SCCC structures. For PCCC structures the interleaver gain is defined by a multiplication factor of N^{-1} in the BER bound. For SCCC structures, the interleaver gain is defined by $N^{-(do+1)/2}$ where do is the free distance of the outer code (e.g., encoder 1 in Figure 6.a). The decoding of serial concatenated codes is similar to parallel concatenated codes by using any of two major soft in soft out (SISO) algorithm MAP algorithm and SOVA algorithm. It can be shown that performance of decoder increase drastically by using iterative decoding .Figure 6.b. shows a typical example of serial concatenated decoder using SIMULINK.

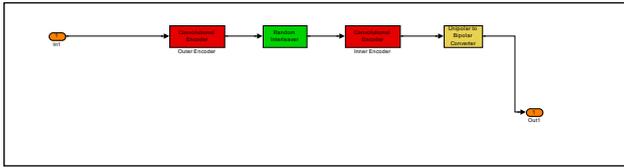


Figure 6. a: Serial Concatenated Encoder

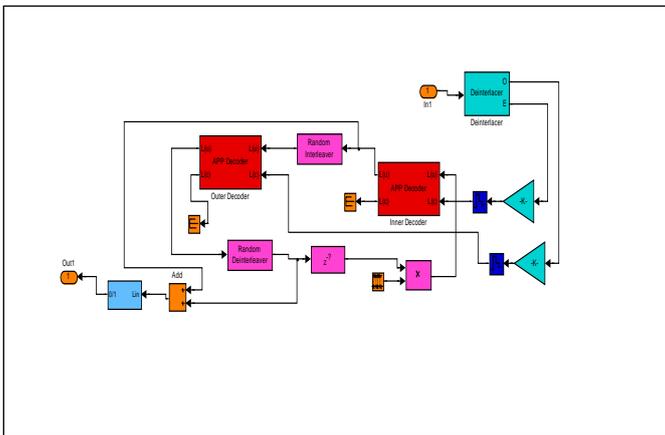


Figure 6.b: Serial Concatenated Decoder

4. DRAWBACK OF PARALLEL AND SERIAL CONCATENATED CODES

Performance of serial and parallel concatenated convolutional schemes with iterative decoding techniques for different interleaver designs were investigated in [21, 22]. Figure 6 shows initial results obtained for the parallel and serial concatenated schemes, respectively, based on inner and outer convolutional codes, and 8 iterations.

For the parallel concatenated scheme illustrated in Figure 7, both the inner and outer codes are identical rate 2/3 16 state RSC codes. For the serial concatenated scheme, the outer code is the same RSC code as used in the parallel scheme, while the inner code is a rate 3/4 16 state RSC code. Serial and parallel schemes having the same delay (60, 600, 6000) are compared. It is characteristic for PCCC schemes to perform better than SCCC schemes at low SNRs. However, increasing the SNR, SCCC schemes outperform PCCC schemes. The cross-over point depends on the interleaver size and interleaver design.

So the final conclusion is that PCCCs perform exceptionally well at low signal-to-noise ratios (SNRs) but develop rather high error floors at high SNRs [3]. On the other hand, SCCCs can achieve extremely low bit error rates at high SNRs, although this comes at the cost of worse performance (relative to PCCCs) at very low SNRs [4].

5. HYBRID CONCATENATED CODES

A hybrid concatenated code with two interleavers is the parallel concatenation of an encoder, which accepts the permuted version of the information sequence as its input, with a serially concatenated code, which accepts the unpermuted information sequence. Hybrid concatenated code is first proposed by Divsalar-Pollara [23], However performance of hybrid concatenated code is further improve by using modified Log –MAP algorithm proposed by Ya-Cheng Lu, Erl-Huei Lu [24]. In modified Log–MAP algorithm both the *extrinsic information* of systematic bits and parity bits can be retrieved during iterative decoding. However at lower SNR condition performance of Divsalar proposed HCCC structure is performed better compared to modified Log-Map algorithm. In this paper

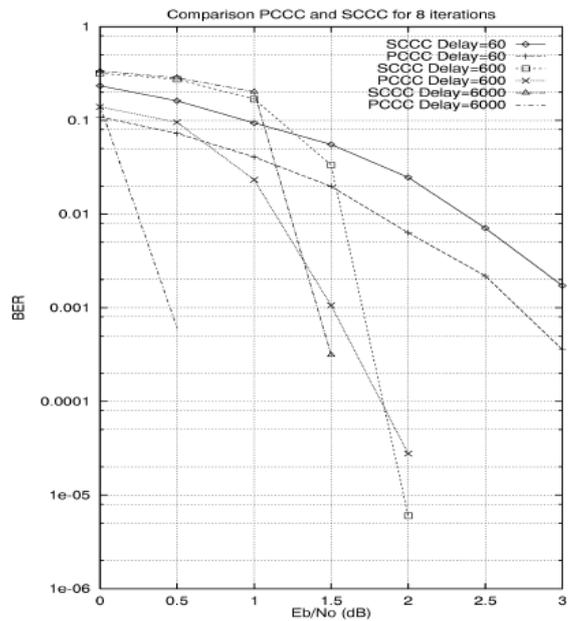


Figure 7: PCCC and SCCC Comparison

we will discuss the performance of Divsalar proposed HCCC structure As for the SCCC structures, the interleaver gain for the HCCC depends on the free distance of the outer code in the serial concatenation part of the HCCC. The multiplication factor is N^{-d_o} for the BER bound, where d_o is the free distance of the outer code. Therefore the HCCC structure is a further improvement on the SCCC structure.

As per proposed structure by Divsalar-Pollara Figure 8.a and 8.b shows the SIMULINK model of hybrid concatenated encoder and decoder

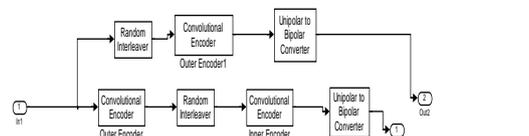
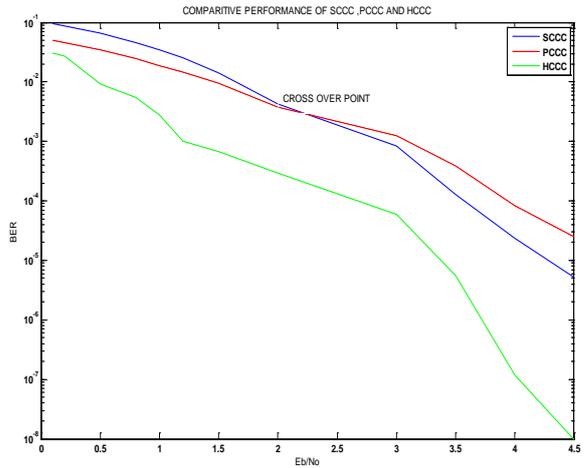


Figure 8.a. Hybrid Concatenated Encoder



Figure 9: Comparative Performance Analysis of PSCC,SCCC



and HSCC Codec
Figure 10: Comparative Performance of Concatenated Codes

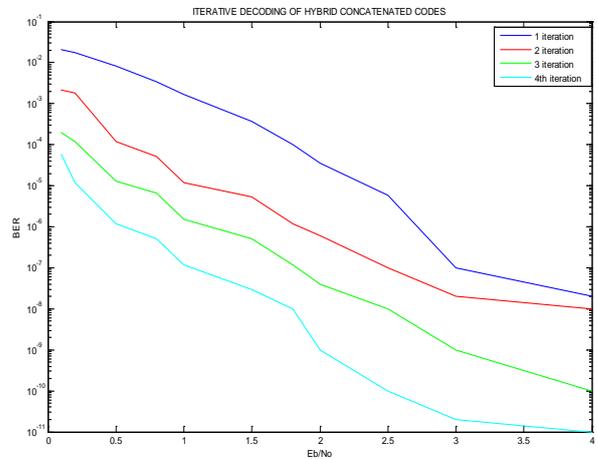


Figure 11: Performance Evaluation of Hybrid Concatenated Codes with respect to Number of Iteration

TYPE	Coding gain at 10 ⁻⁵ (dB)	Coding gain at 10 ⁻⁶ (dB)	Initial decoding Delay(ms)(200MHz)	Performance
SCCC	7.6	9.2	5.00	Perform better at Higher Eb/No
PCCC	8.0	8.4	3.9	Perform better at Lower Eb/No
HCCC	9.00	9.8	6.50	Perform better Lower as well as Higher Eb/No

*Result based on 3 Iteration in all cases.

TABLE-1

It can be easily seen that as the number iteration increases the performance of HCCC concatenated code increases drastically. However as the number of iteration increases the decoding delay and complexity increase also increase proportionally.

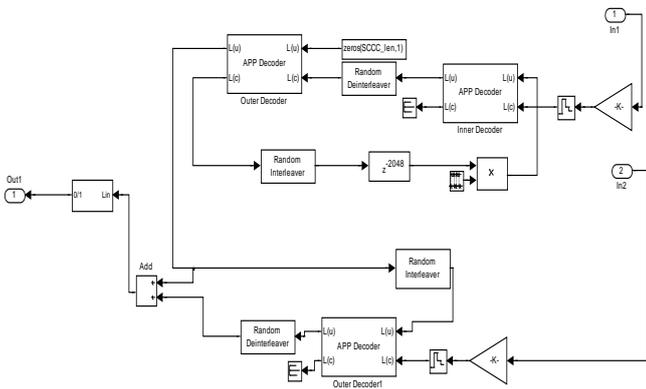
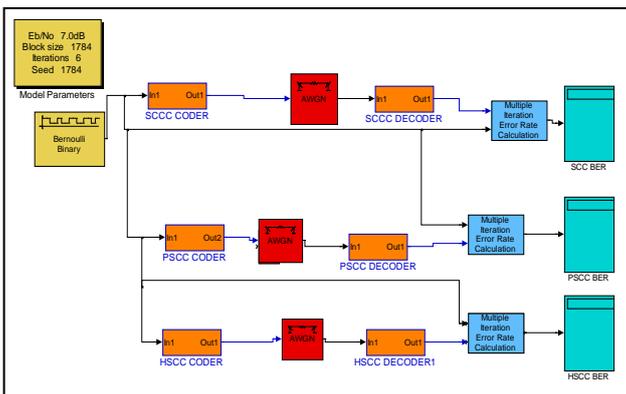


Figure 8.b. Hybrid Concatenated Decoder

6. SIMULATION RESULT

Figure 9 shows the comparative performance of all three types of concatenated code structure. The simulation parameter kept identical in all three cases. The code rate for all three cases is 1/2 and total decoding iteration is set to 6. The frame length is set to 1784 Bits. In all three cases random interleaver and deinterleaver is used. The comparative performance of all three type structure is shown in Figure 10.

The comparative performance in terms of coding gain with QPSK modulation scheme is mention in Table 1. It can be easily seen that the serial concatenated performance is superior compare to parallel concatenated code at higher Eb/No, while at lower Eb/No performance of parallel concatenated code is better. It may be noted that cross over point of SCCCs and PCCCs is depend upon frame length, type of interleaver and number of iteration. The performance of hybrid concatenated code is superior at all Eb/No. Hence hybrid concatenated code is the suitable candidate for deep space mission. The key point in the performance of all three codes is iterative decoding. In order to verify how number of iteration will improve the performance Figure 11 will shows the performance of hybrid concatenated code with respect to number of iteration.



Now we know that selection of interleaver also play a vital role in the performance of concatenated codes. Figure 12 show comparative performance analysis of HCCC code using random, matrix, helical and circular interleaver. Table 2 shows the error free performance analysis of HSCC codec with respect to different interleavers. It can be easily seen that performance of random interleaved HSCC codec is superior compare to other interleaved HSCC codec. Hence random interleaver is the suitable choice for HSCC codec.

Table: 2 Packet size : 1784 Bits, BER=10⁻⁶

S.No	Interleaver Type	1 st iteration (Eb/No)	2 nd iteration (Eb/No)	3 rd iteration (Eb/No)
1	Pseudo random	4.4	3.3	2.1
2	Matrix	5.5	4.8	3.9
3	Helical	5.2	4.7	3.8
4	Circular	5.6	5.0	4.3
5	Algebraic	5.1	4.2	3.5

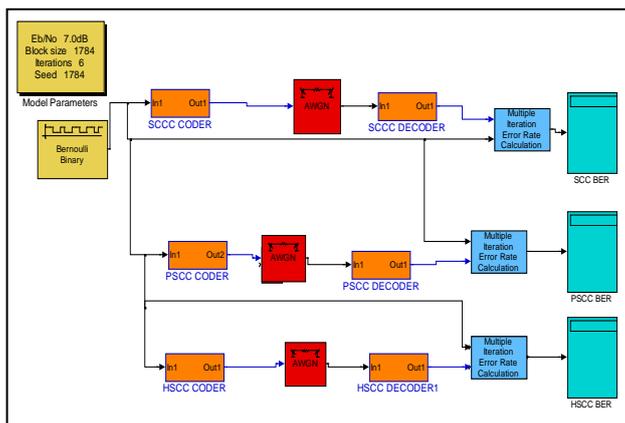


Figure 12: Comparative Performance Analysis of Hybrid Concatenated Codes with respect to different Interleavers

CONCLUSION

A detailed simulation result are presented for all three concatenated code structure for ISRO deep space mission . Simulation result shows that for identical code rate, the performance of Hybrid concatenated code is superior compare to serial and Parallel concatenated code structure. However complexity of Hybrid concatenated decoder is higher compare to other concatenated code structure. Simulation result also shows that random interleaver is the ideal choice for hybrid concatenated code structure. Hence

random interleaved Hybrid concatenated code is suitable candidate for ISRO deep space mission.

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